Option-Based Range Estimator and Application

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1 Introduction

Volatility is a core concept in modern finance theory, including in asset pricing, portfolio allocation or risk management. Literature has evolved from a constant volatility (Merton, 1969; Black and Scholes, 1973) to a time-varying fashion (e.g, Andersen and Bollerslev, 1997), and stochastic volatility models are also widely used. Stochastic volatility models are heavily used in both academia and industry (e.g., Hull and Whilte, 1987; Heston, 1993; Bates, 1996; Ghysels, Harvey, and Renault, 1996; Jarrow, 1998; Duffie, Pan, and Singleton, 2000). However, it is not easy to estimate the stochastic volatility model. For instance, the Gaussian quasi-maximum likelihood estimation (QMLE) approach of Harvey, Ruiz, and Shephard (1994), seems appealing due to its simplicity. Nevertheless, the problem embedded in these line of estimation is that standard volatility proxies such as squared returns are contaminated by highly non-Gaussian error.

Among financial practitioner, it is very useful if we can know the future distribution in advance, such as VaR (Value-at-Risk) analysis, expected shortfall (ES) for risk management, break-even point for option traders. The classical way to estimate the return distribution or interval rely on the estimation of volatility, then transfer to the confidence interval with an assumption on the distribution (Normality, for example). Nevertheless, returns are not normally distributed, forecast return confidence interval through volatility is thus biased.

In the last decades, a number of range based estimators have been proposed. Price range, which is defined as the difference in th high and low price observed during a time interval, is more efficient than return based estimators (e.g., Rogers and Satchell, 1991; Alizadeh, Brandt and Diebold, 2002 and Bali and Weinbaum 2005). However, range based estimators are proposed with assumption that the price dynamic is continuous and lognormally distributed. These estimators may be more efficient and unbiased than return-based estimators, however, it can be the case in reality that the prices are not observed continuously.

There is a growing literature about extracting information from option market to forecast asset returns and volatilities. Bakshi and Madan (2000) propose a model-free method to estimate the risk-neutral moments of underlying from option prices. For instance, Conrad, Dittmar and Ghysels (2013) use option prices to estimate ex ante higher moments of the underlying individual securities risk-neutral returns distribution. They find that securities risk-neutral volatility, skewness, and kurtosis are strongly related to future returns. Many studies have demonstrated that option-implied volatility is a strong predictor of the future volatility in equity markets. Classic contributions include Christensen and Prabhala (1998), and Blair, Poon, and Taylor (2001). The predictive power of option-implied equity volatility has been confirmed recently by Busch, Christensen and Nielsen (2008), who compare option-implied forecasts with state-of-the-art realized volatility forecasts.

In this paper, I propose a ranger estimator similar to range-based volatility estimator, with intraday minute data. Then, I use VIX data as variable to forecast the future realized range, the correlation for the whole period is 0.73, while the R-square is 0.55, indicating a very well out-of-sample fitting. It also dominates historical based volatility estimation method.

2 Range Estimator

It is very useful for financial practitioners, if they can know the future distribution in advance, such as VaR (Value-at-Risk) analysis, expected shortfall (ES) for risk management, break-even point for option traders. For lots of option trading strategies, underlying price confidence interval is critical, for instance, choosing different call and put to short a strangle requires the bet/believe on the break-even point, in another word, the confidence interval. This knowledge might not be so interesting for a stock trader, however, for derivative traders (who are dealing with more sophisticated instruments), they are profitable. In addition, people in risk management area are also interested in price/return distribution, in their word, tail-risk. The philosophy of risk management lies in the Black-Swan, historically, those low probability events crash the industry, and cause significant loss. Thus, with the knowledge of future distribution, risk management team could produce more reliable numbers. The classical way to estimate the return distribution or interval rely on the estimation of volatility, then transfer to the confidence interval with an assumption on the distribution (Normality, for example). Nevertheless, returns are not normally distributed, forecast return confidence interval through volatility is thus biased.

2.1 Range Estimator Construction

Denote P_0 the price of the security at time t_0, P_i the price of the security at time t_i, where $i = 1, 2, ...N$. For a given period (can be a day, a week or a month depending on the data frequency), denote the range of the price dynamics with respect to the initial price P_0:

$$C_i = \frac{P_i}{P_0}, i = 1, 2, ...N \tag{1}$$

Sort C_i from low to high, for a given confidence level α, calculate the confidence interval $C^{HL} = C^{sorted}_{1-\alpha} - C^{sorted}_{\alpha}$, for instance, if we want to calculate the 90% confidence interval for the future distribution, we can calculate the difference between 95 percentile sorted value and 5 percentile sorted value, $C^{HL}_{90\%}{=}C^{sorted}_{95\%} - C^{sorted}_{5\%}$. Compare to the classic range estimator, which usually defined as the difference of lowest and highest value, a confidence interval could be less affected by the extreme values—- the sudden change in the market (might recover very soon), indicates a more fruitful information for traders.

2.2 Data and Empirical Results

The data are minutes data of SPY (S&P500 ETF) price, range from 2010.06 to 2017.05. For each trading day, we have 405 observations. The data is from CBOE.

[insert Figure 1 here]

[insert Table 2 here]

Figure 1 plots the $80\%, 90\%, 95\%$ and 99% percentile PREs for one week distribution. As the figure shows, it hits high level during second half of

2011, when there are fears of contagion of the European sovereign debt crisis to Spain and Italy, as well as concerns over France's current AAA rating, concerns over the slow economic growth of the United States and its credit rating being downgraded. Among different percentile PREs, the pattern is quite similar, correlations between different PREs are almost 1, indicating its consistence. Table 1 summarize the statistics for the PREs with different percentiles. The means and medians are increasing monotonically as expected. Results of skewness and kurtosis showing the distribution of PREs are highly non-normal. For instance, the mean of PRE with 99 percentile is 0.0234, it indicates on average, the 99% of the one week SPY minute price is deviated within 2.34% of the SPY Monday open price (defined as the S_0).

2.3 Correlation with VIX

There is a growing literature about extracting information from option market to forecast asset returns and volatilities. Bakshi and Madan (2000) propose a model-free method to estimate the risk-neutral moments of underlying from option prices. Classic contributions include Christensen and Prabhala (1998), and Blair, Poon, and Taylor (2001). The predictive power of option-implied equity volatility has been confirmed recently by Busch, Christensen and Nielsen (2008), who compare option-implied forecasts with state-of-the-art realized volatility forecasts. Option, especially OTM (out-of-the-money option) contains unique information about the market expectation of the underlying future movements. When investors/institution traders perceive (or know in advance due to asymmetric information) that certain event is going to happen, we could observe money flows into OTM options, due to the high leverage effect. These money flows are then push the IV (implied volatility) to a higher level, in the end, affect the VIX. One could take the advantage of observing VIX to get aggregate market "emotion". Consequently, it is interesting to see whether/how these information can forecast future.

[**insert table 3 here**]

Figure 3 plots the Monday VIX open price with the subsequent week price distribution, with different percentile. The correlations of VIX price with different percentile distribution remains high for the whole period, range from 0.65 to 0.75, indicate that VIX is indeed a forward-looking measurement, at least for this percentile range estimator. Its correlation is actually increase as the percentile increase, showing the ability to capture/forecast the tail risk.

3 Forecast Percentile Range

There is no doubt that future distribution information is crucial in the finance world, both for academia and industry. The percentile range estimator (PRE) could be applied in both trading and risk management, thus a decent forecast method is appealing. The classical method is to first estimate the volatility, then transfer to the confidence interval based on the assumption of the distribution (normality, for example). However due to highly non-Gaussian feature, this kind of transformation causes biased estimator. In this section, I propose a new simple regression based model to fit and forecast FPE, other benchmark methods are also used for comparison.

3.1 VIX as the variable to estimate PRE

In the previous section, I show that the realized percentile range is highly correlated with VIX price, to be more precise, the one week realized percentile range is over 70% correlated with the Monday open price of VIX. Naturally, given this level of correlation, we can use a regression model to estimate the relationship, and forecast accordingly. I use OLS to estimate the following regression,

$$C^{HL}_{\alpha,t,T} = C + b1 \cdot VIX_t + b2 \cdot VIX_{t-1} \tag{2}$$

Table 2 summarize the statistic results and R-square for the fitted values. The fitted PREs also show high non-normality, with skewness up to 2 and kurtosis is around 8. Both R squares and adjusted R squares are at high level, more than 50% for all percentiles, indicate very good model fit. Interest to notice, the R-square increases as the chosen percentile increases, showing the potential to capture the edge cases.

3.2 Forecast PRE using VIX

In the previous section, the results showing that with VIX variable (and lagged one), the linear model could provide very good fitness. In order to forecast PRE, we need ex-ante coefficients, I use moving window estimation strategy, as its popularity among the literature. The coefficients are estimated using backward 256 weeks observation (approximately 5 years to generate stable estimates):

$$C^{HL,forecast}_{\alpha,t,T} = C_{t-256,t} + b1_{t-256,t} \cdot VIX_t + b2_{t-256,t} \cdot VIX_{t-1}. \tag{3}$$

3.3 Alternative Methods

The alternative method is to estimate the volatility and then transfer to distribution interval with an assumption on the distribution itself (usually normality). As a benchmark method, I use rolling window to estimate historical volatility, then apply the normality assumption, a 90% distribution interval is equal to $\mu \pm 1.5\sigma$. We can further assume that the weekly mean of deviation from the original price (S_0) is 0, then the 90 percentile range estimator is equal to 3σ. Figure 6 plots the comparison between VIX based forecast PREs, volatility transferred based PREs and realized PREs. As the figure shows, the volatility transferred based method generates biased estimates, they are over estimated through the whole time period. Another method can be using realized volatility (RV) and/or PRE_{t-1} as variables. Table 4 summarizes the panel regressions for estimating PRE_t. PRE_t is the realized PRE (Percentile Range Estimator) for the week (Monday to Friday), VIX_t stands for the Monday open price of VIX index. VIX_{t-1} is the last Monday (t-1) open price of VIX index. RV_{t-1} is the last week realized volatility calculating using minute data. PFE_{t-1} stands for the realized PRE of last week. It shows that the both historical and option-implied methods work well alone, although the fitting performance of using VIX_t and VIX_{t-1} as variables is much better than the historical ones. The $adj.R^2$ of Regression (1) is more than doubled comparing to Regression (2) and (3). This is intuitive, since we believe the VIX index price on Monday should not only reflect the performance of S&P500 during last week, but also absorb the information during weekends, to form a rational expectation of the market for the following week. Interestingly, if we put historical metrics RV_{t-1} and PFE_{t-1} together with the option-implied ones (VIX_t, VIX_{t-1}) in Regression (5) and (6), the $adj.R^2$s do not increase, RV_{t-1} and PFE_{t-1} are now insignificant, showing not value is added by incorporating historical metrics.

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Appendices

A Appendix: Model Free Option-Implied Moments

Bakshi, Carr and Madan (2000) show that any twice continuously differentiable fnct, $H(S_T)$, of terminal price S_T, can be replicated by a unique position in the risk-free, stocks and European options.

$$H[S] = H[\bar{S}]+(S-\bar{S})H_s[\bar{S}]+\int_{\bar{S}}^{\infty} H_{ss}[K](S-K)^{+}dK+\int_{0}^{\bar{s}} H_{ss}[K](K-S)^{+}dK \tag{4}$$

The prices of these contracts are

$$\begin{aligned} &E_t^Q\{e^{-r\tau}H[S]\} = (H[\bar{S}] - \bar{S}H_s[\bar{S}])e^{-r\tau} + H_s[\bar{s}]S(t)+ \\ &\int_{\bar{s}}^{\infty} H_{ss}[K]C(t,\tau;K)dK + \int_{0}^{\bar{S}} H_{ss}[K]P(t,\tau;K)dK. \end{aligned} \tag{5}$$

where $C_t(\tau, K)$ and $P_t(\tau, K)$ are prices of the European call and put options with time to maturity τ and strike price K. As a result, we can calculate the prices of derivatives given the price of the risk free zero coupon bond r, the spot price of the underlying, $\bar{S}$, and a series of OTM calls and puts. Since our main interest would be underlying return distribution, consider the function $H[S]$):

$$H[S_{t+\tau}] = R_{t+\tau}^2 = (lnS_{t+\tau} - lnS_t)^2 \tag{6}$$

Then the risk-neutral variance, skewness and kurtosis of equity returns could be computed based on the following expressions.

$$E_0^Q\Big[e^{-rT}(\frac{S_T-S_0}{S_0})^2\Big] = \frac{2}{s_0^2}\Big[\int_0^{S_0} P_0(T,X)dX + \int_{S_0}^{\infty} C_0(T,X)dX\Big] \tag{7}$$

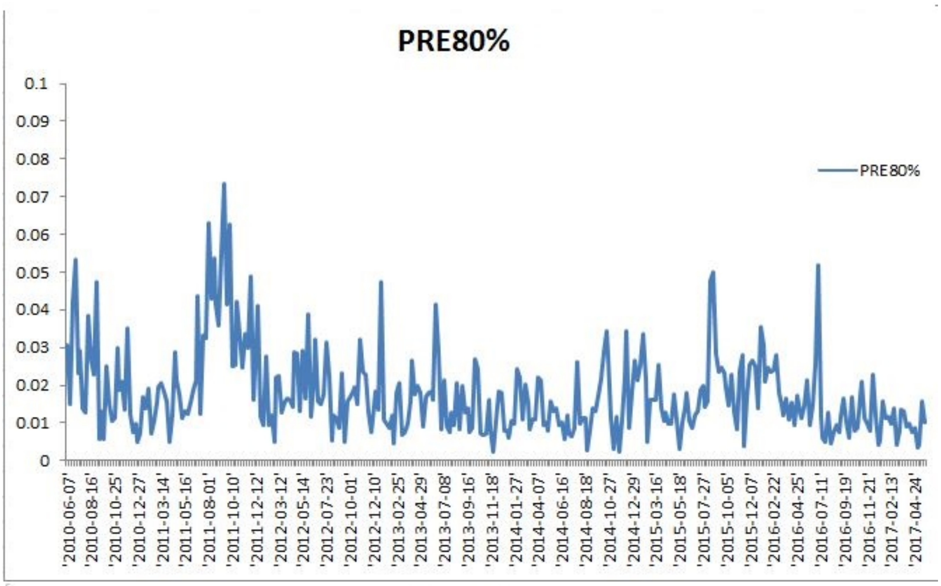
$$E_0^Q\Big[e^{-rT}(\frac{S_T-S_0}{S_0})^3\Big] = \frac{6}{S_0^2}\Big[\int_0^{S_0} (\frac{X-S_0}{S_0})P_0(T,X)dX + \int_{S_0}^{\infty} (\frac{X-S_0}{S_0})C_0(T,X)dX\Big] \tag{8}$$

$$E_0^Q\Big[e^{-rT}(\frac{S_T-S_0}{S_0})^4\Big] = \frac{12}{S_0^2}\Big[\int_0^{S_0} (\frac{X-S_0}{S_0})^2P_0(T,X)dX + \int_{S_0}^{\infty} (\frac{X-S_0}{S_0})^2C_0(T,X)dX\Big] \tag{9}$$

Since there is no continuity of strike prices, we can approximate the integrals using cubic spline. For a given maturity, I interpolate implied volatilities across different moneyness level (K/S) to obtain a continuum of implied volatilities. Furthermore, the implied volatility of the highest or lowest available strike price is used when moneyness below and above the available moneyness level in the market. More precisely, for moneyness level smaller than 1 ($K/S < 1$), the corresponding implied volatilities are used to generate put option prices, while for moneyness level larger than 1 ($K/S < 1$), the corresponding implied volatilities are used to generate call option prices.

Figure 1: Percentile Range Estimator

Figure 1 plots the 90% percentile range estimator for one week SPY distribution. Denote P_0 the price of the security at time t_0, P_i the price of the security at time t_i, where $i = 1, 2, ...N$. For a given period (can be a day, a week or a month depending on the data frequency), denote the range of the price dynamics with respect to the initial price P_0: $C_i = \frac{P_i}{P_0}, i = 1, 2, ...N$. Sort C_i from low to high, for a given confidence level α, calculate the confidence interval $C^{HL} = C^{sorted}_{1-\alpha} - C^{sorted}_{\alpha}$, for instance, if we want to calculate the 90% confidence interval for the future distribution, we can calculate the difference between 95 percentile sorted value and 5 percentile sorted value, $C^{HL}_{90\%} = C^{sorted}_{95\%} - C^{sorted}_{5\%}$. The data are minutes data of SPY (S&P500 ETF) price, range from 2010.06 to 2017.05. For each trading day, we have 405 observations. The data is from CBOE.



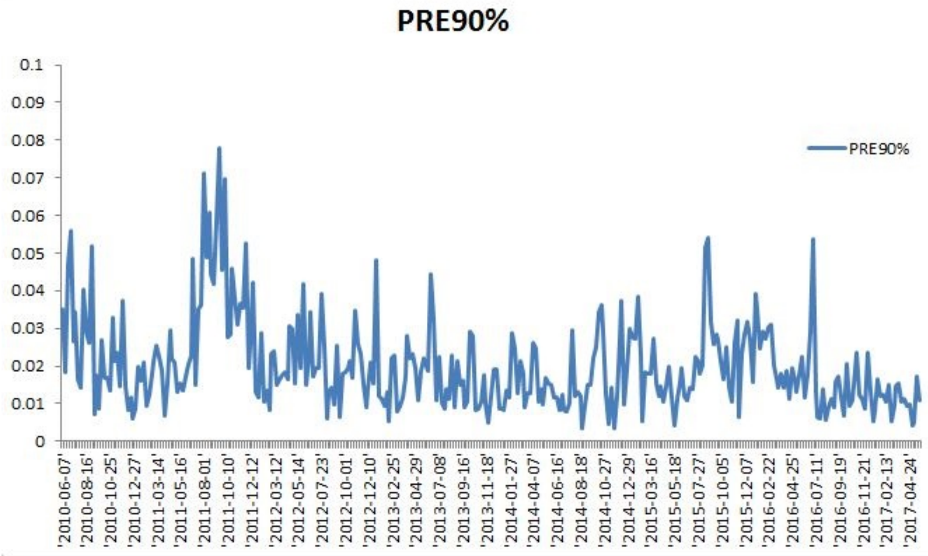


Table 1: Summary Statistics for PRE

Table 1 shows the different percentile range estimators for one week SPY distribution. Denote P_0 the price of the security at time t_0, P_i the price of the security at time t_i, where $i = 1, 2, ...N$. For a given period (can be a day, a week or a month depending on the data frequency), denote the range of the price dynamics with respect to the initial price P_0: $C_i = \frac{P_i}{P_0}, i = 1, 2, ...N$. Sort C_i from low to high, for a given confidence level α, calculate the confidence interval $C^{HL} = C^{sorted}_{1-\alpha} - C^{sorted}_{\alpha}$, for instance, if we want to calculate the 90% confidence interval for the future distribution, we can calculate the difference between 95 percentile sorted value and 5 percentile sorted value, $C^{HL}_{90\%}$=$C^{sorted}_{95\%} - C^{sorted}_{5\%}$. The data are minutes data of SPY (S&P500 ETF) price, range from 2010.06 to 2017.05. For each trading day, we have 405 observations. The data is from CBOE.

	PRE(80%)	PRE(90%)	PRE(95%)	PRE(99%)
mean	0.0178	0.0201	0.0218	0.0234
median	0.0149	0.0173	0.0188	0.0201
volatility	0.0113	0.0121	0.0128	0.0135
skewness	1.6364	1.6065	1.6145	1.6917
kurtosis	6.4365	6.3578	6.3623	6.8715

Table 2: Summary Statistics for fitted PRE

Table 2 reports the summary statistics for fitted percentile range estimator (PRE) for one week SPY distribution with different percentiles. PRE is defined in the following way: Denote P_0 the price of the security at time t_0, P_i the price of the security at time t_i, where $i = 1, 2, ...N$. For a given period (can be a day, a week or a month depending on the data frequency), denote the range of the price dynamics with respect to the initial price P_0: $C_i = \frac{P_i}{P_0}, i = 1, 2, ...N$. Sort C_i from low to high, for a given confidence level α, calculate the confidence interval $C^{HL} = C^{sorted}_{1-\alpha} - C^{sorted}_{\alpha}$. The fitted PREs are estimated through regression $C^{HL}_{\alpha,t,T} = C + b1 \cdot VIX_t + b2 \cdot VIX_{t-1}$. The data are minutes data of SPY (S&P500 ETF) price, range from 2010.06 to 2017.05. For each trading day, we have 405 observations, which constitute 2025 observations per week. The data is from CBOE.

	PRE(80%)	PRE(90%)	PRE(95%)	PRE(99%)
mean	0.0181	0.0202	0.0218	0.0233
median	0.0154	0.0174	0.0188	0.0201
volatility	0.0089	0.0094	0.01	0.0104
skewness	2.1785	2.1639	2.1496	2.1406
kurtosis	8.3723	8.347	8.3232	8.3185
R square	0.503	0.5296	0.5453	0.5566
adj. R^2	0.5131	0.5281	0.544	0.5553

Table 3: My caption

Table 3 reports the summary statistics for fitted percentile range estimator (PRE) for one week SPY distribution with different percentiles. PRE is defined in the following way: Denote P_0 the price of the security at time t_0, P_i the price of the security at time t_i, where $i = 1, 2, ...N$. For a given period (can be a day, a week or a month depending on the data frequency), denote the range of the price dynamics with respect to the initial price P_0: $C_i = \frac{P_i}{P_0}, i = 1, 2, ...N$. Sort C_i from low to high, for a given confidence level α, calculate the confidence interval $C^{HL} = C_{1-\alpha}^{sorted} - C_{\alpha}^{sorted}$. The fitted PREs are estimated through regression $C_{\alpha,t,T}^{HL} = C + b1 \cdot VIX_t + b2 \cdot VIX_{t-1}$. The data are minutes data of SPY (S&P500 ETF) price, range from 2010.06 to 2017.05. For each trading day, we have 405 observations, which constitute 2025 observations per week. The data is from CBOE.

	PRE(80%)	PRE(90%)	PRE(95%)	PRE(99%)
mean	0.0179	0.0201	0.0216	0.023
median	0.0159	0.0179	0.0192	0.0205
volatility	0.0075	0.0082	0.0086	0.009
skewness	1.9272	1.9187	1.8908	1.8808
kurtosis	7.1501	7.0968	6.9574	6.9656
MSE	0.0205	0.0224	0.0253	0.0295

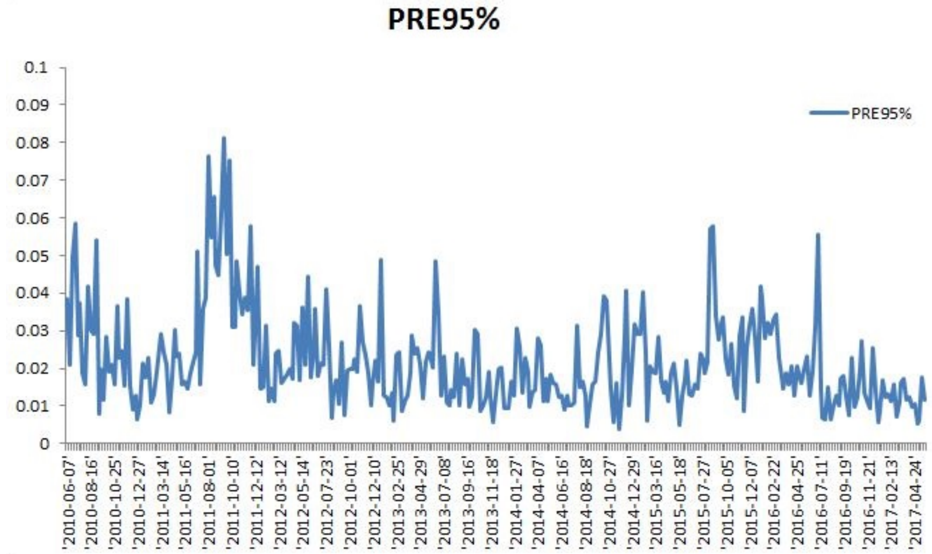
Table 4: Table 4: Panel Regressions for Different Variables

Table 4 summarizes the panel regressions for estimating PRE_t. PRE_t is the realized PRE (Percentile Range Estimator) for the week (Monday to Friday), VIX_t stands for the Monday open price of VIX index. VIX_{t-1} is the last Monday (t-1) open price of VIX index. RV_{t-1} is the last week realized volatility calculating using minute data. PFE_{t-1} stands for the realized PRE of last week. PRE is defined in the following way: Denote P_0 the price of the security at time t_0, P_i the price of the security at time t_i, where $i = 1, 2, ...N$. For a given period (can be a day, a week or a month depending on the data frequency), denote the range of the price dynamics with respect to the initial price P_0: $C_i = \frac{P_i}{P_0}, i = 1, 2, ...N$. The data are minutes data of SPY (S&P500 ETF) price, range from 2010.06 to 2017.05. For each trading day, we have 405 observations, which constitute 2025 observations per week. The data is from CBOE.

	$PRE_t(80\%)$					
	(1)	(2)	(3)	(4)	(5)	(6)
VIX_t	0.180***				0.158***	0.180***
	(11.36)				(12.56)	(11.07)
VIX_{t-1}	-0.039***					-0.045***
	(-2.56)					(-2.41)
RV_{t-1}		0.532***		0.304***	0.084	-0.004
		(8.84)		(4.38)	(-1.29)	(-0.055)
PRE_{t-1}			0.475***	0.334***	0.016	0.054
			(9.77)	(5.81)	(0.29)	(0.98)
observation	328	328	328	328	328	328
adj. Rˆ2	51.3%	19.1%	22.4%	26.5%	50.4%	51.2%

Figure 2: Percentile Range Estimator

Figure 2 plots the 90% percentile range estimator for one week SPY distribution. Denote P_0 the price of the security at time t_0, P_i the price of the security at time t_i, where $i = 1, 2, ...N$. For a given period (can be a day, a week or a month depending on the data frequency), denote the range of the price dynamics with respect to the initial price P_0: $C_i = \frac{P_i}{P_0}, i = 1, 2, ...N$. Sort C_i from low to high, for a given confidence level α, calculate the confidence interval $C^{HL} = C^{sorted}_{1-\alpha} - C^{sorted}_{\alpha}$, for instance, if we want to calculate the 90% confidence interval for the future distribution, we can calculate the difference between 95 percentile sorted value and 5 percentile sorted value, $C^{HL}_{90\%} = C^{sorted}_{95\%} - C^{sorted}_{5\%}$. The data are minutes data of SPY (S&P500 ETF) price, range from 2010.06 to 2017.05. For each trading day, we have 405 observations. The data is from CBOE.



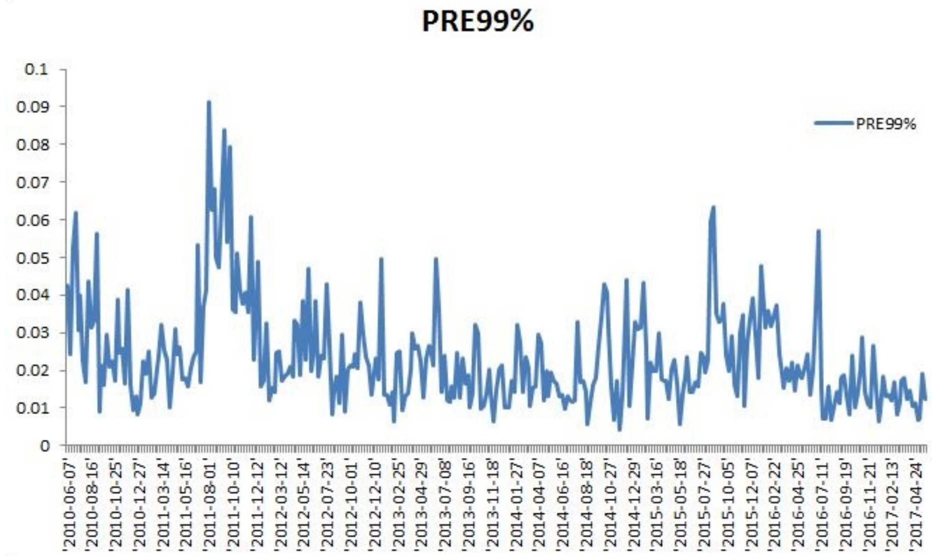


Figure 3: Percentile Range Estimator

Figure 7 plots the 90% percentile range estimator for one week SPY distribution and Monday VIX open price. The correlation between VIX price and subsequent realized PRE is also given. The data are minutes data of SPY (S&P500 ETF) price, range from 2010.06 to 2017.05. For each trading day, we have 405 observations. The data is from CBOE.

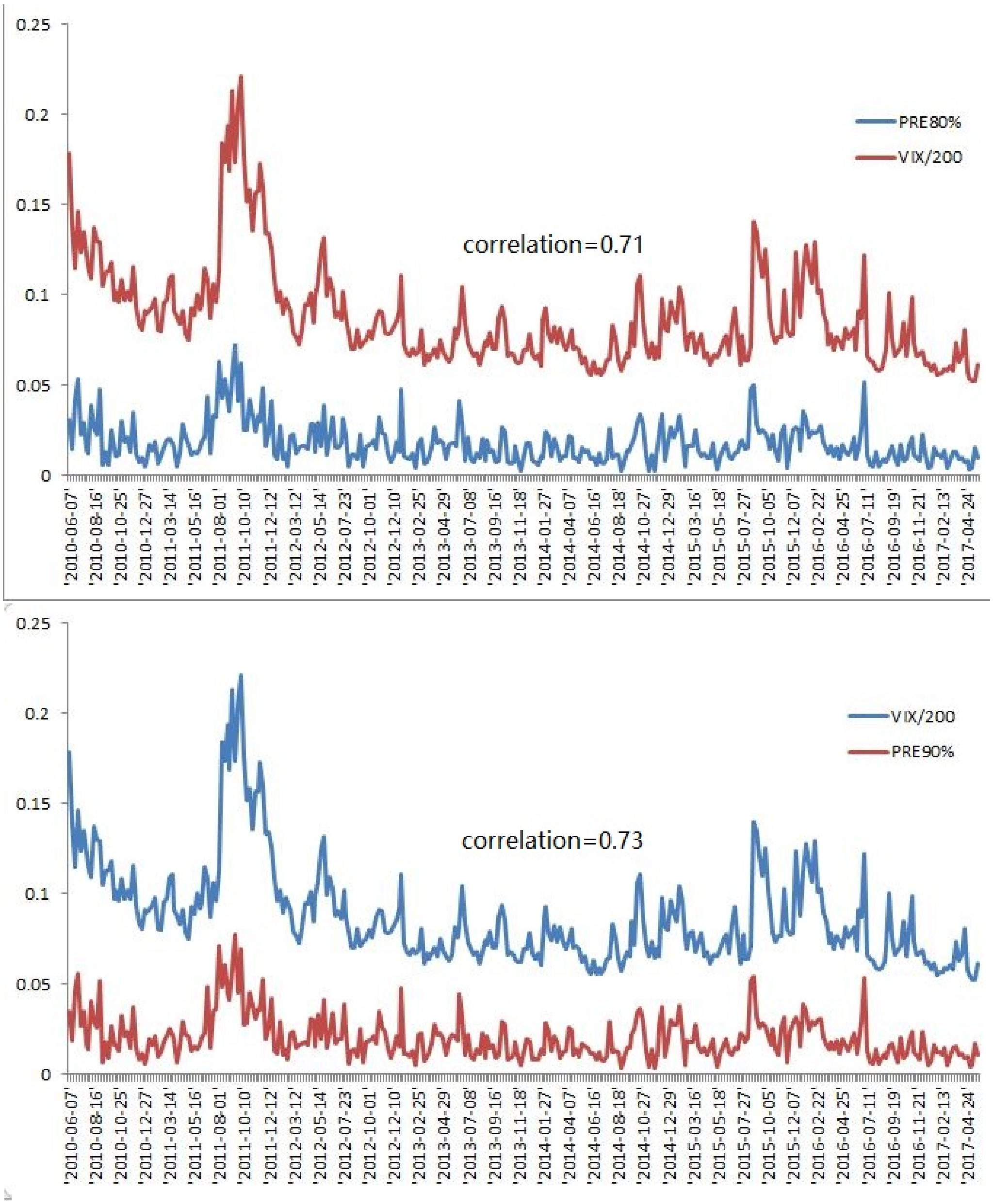


Figure 4: Percentile Range Estimator

Figure 6 plots the 90% percentile range estimator for one week SPY distribution and Monday VIX open price. The correlation between VIX price and subsequent realized PRE is also given. The data are minutes data of SPY (S&P500 ETF) price, range from 2010.06 to 2017.05. For each trading day, we have 405 observations. The data is from CBOE.

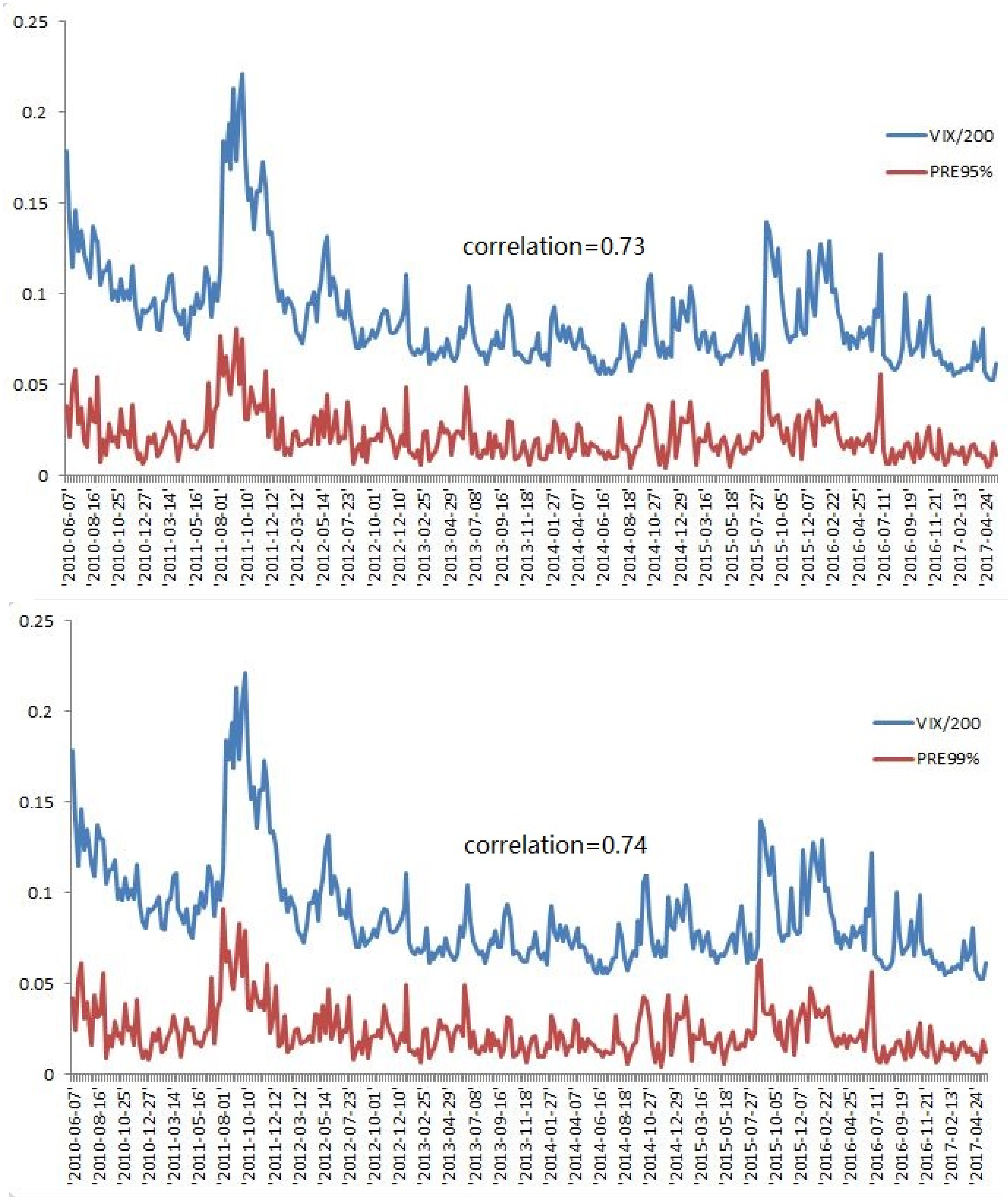
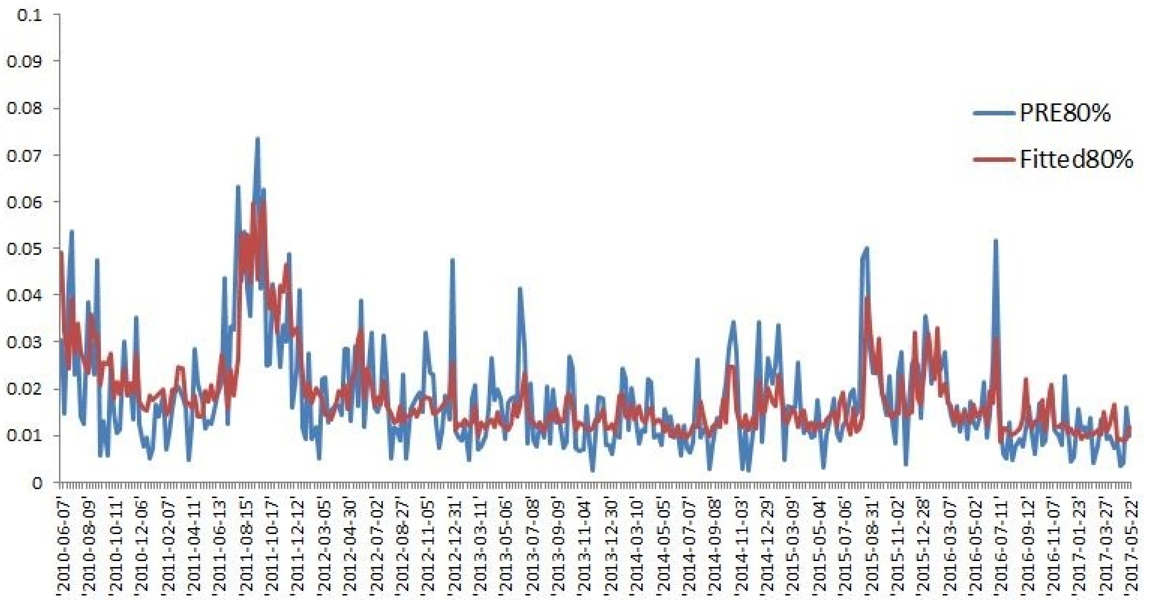


Figure 5: Percentile Range Estimator

Figure 5 plots the fitted percentile range estimator (PRE) for one week SPY distribution and realized PRE. The fitted PREs are estimated through regression $C^{HL}_{\alpha,t,T} = C + b1 \cdot VIX_t + b2 \cdot VIX_{t-1}$ with 80 and 90 percentile during the whole sample period. The data are minutes data of SPY (S&P500 ETF) price, range from 2010.06 to 2017.05. For each trading day, we have 405 observations, which constitute 2025 observations per week. The data is from CBOE.



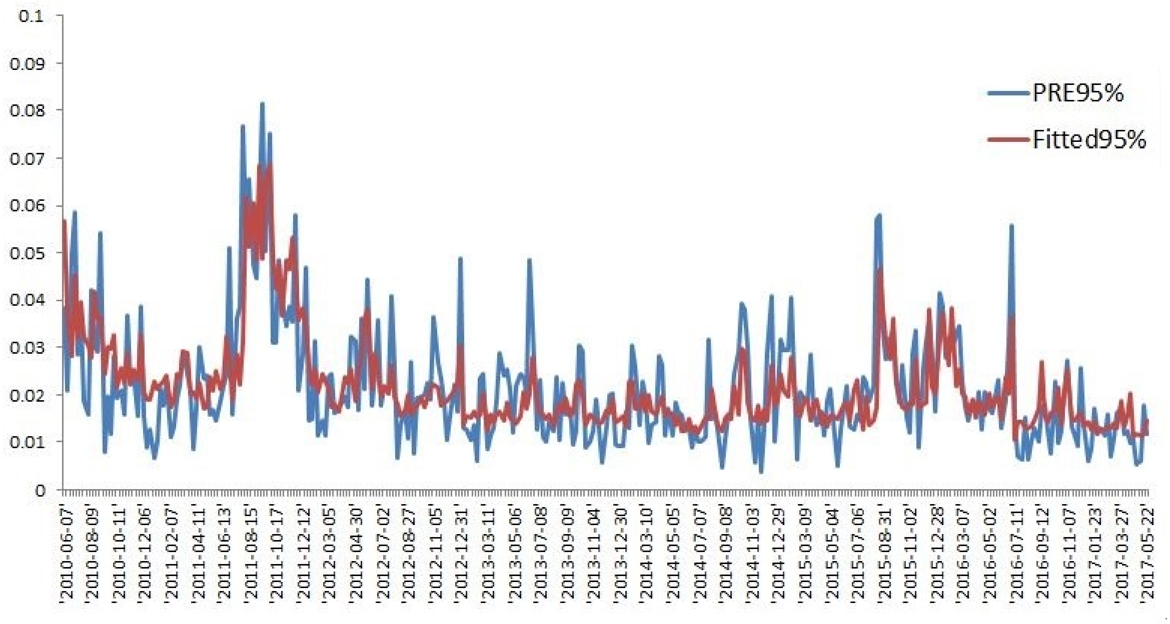


Figure 6: Percentile Range Estimator

Figure 6 plots the fitted percentile range estimator (PRE) for one week SPY distribution and realized PRE. The fitted PREs are estimated through regression $C^{HL}_{\alpha,t,T} = C + b1 \cdot VIX_t + b2 \cdot VIX_{t-1}$ with 80 and 90 percentile during the whole sample period. The data are minutes data of SPY (S&P500 ETF) price, range from 2010.06 to 2017.05. For each trading day, we have 405 observations, which constitute 2025 observations per week. The data is from CBOE.