The Cross-Section of Equity Option Returns

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Abstract

Empirical research has argued that option returns are anomalous based on standard return metrics, such as average returns or Sharpe ratios. Other studies treat this return anomaly as evidence that skewness preference is priced. Recent theoretical developments predict a negative relationship between total skewness and average returns. Based on the newly developed β-Heston model, I study the cross-section of equity option returns to investigate the out-of-the-money option mispricing issue. I find that by comparing historical statistics to those generated by the model, the puzzling out-of-the-money put returns are consistent with the β-Heston model estimation. I also find that the well documented skewness preference is not priced in equity options. Additionally, I provide evidence that casts doubt on the hypothesis of market imperfections and constrained financial intermediaries.

1 Introduction

Recent studies have concluded that options are mispriced in the sense that certain option returns are excessive relative to their risks. For instance, Bondarenko (2003) reports that average at-the-money (ATM) put returns are -40% per month, and deep out-of-the-money (OTM) put returns are -95% per month for the S&P 500 index. Furthermore, standard return-based measures such as CAPM alphas or Sharpe ratios are statistically significant and larger than those of the underlying index.

However, we should pay attention to certain conditions when applying these metrics. Option returns are highly non-normal, and these metrics assume normality, which is inappropriate. Additionally, average put returns should be negative due to the leverage inherent in options and the presence of higher moment risk premium. To alleviate these issues, Broadie, Chernov, and Johannes (2009) use option pricing models as a benchmark to assess evidence for index option mispricing. They find that average returns, CAPM alpha, and Sharpe ratios for deep OTM put returns are statistically insignificant when compared to the Black-Scholes model.

Another strand of the literature treats this mispricing (overpriced put options) phenomenon as evidence that skewness preference is priced. Recent studies show that standard rational asset pricing models have difficulty explaining many of the basic empirical facts about financial markets. Experimental economists find that individuals deviate from standard utility theory when making choices in the face of uncertainty. For instance, investors prefer skewness or lottery-like features in asset return distributions, and these preferences influence asset prices in equilibrium. Based on these theories, Boyer and Vorkink (2014) find that total skewness is priced: portfolios of short-term options with high ex ante skewness lose approximately 10% to 50% per week on average compared to those with low ex ante skewness. Bali and Murray (2013) investigate the pricing of risk-neutral skewness in the stock options market by creating skewness assets (comprised of options and underlying equities). They find a strong negative relation between risk-neutral skewness and asset returns, which is consistent with a positive skewness preference.

Cao and Han (2012) present a robust finding that delta-hedged equity option return decreases monotonically with an increase in the idiosyncratic volatility of the underlying stock. The intuition behind this finding relates to market imperfections and constrained financial intermediaries: dealers charge a higher premium for options with high idiosyncratic volatility of the underlying stock due to their higher arbitrage costs. This hypothesis is motivated by the theory of option pricing in an imperfect market that emphasizes the role of constrained financial intermediaries. Shleifer and Vishny (1997) argue that idiosyncratic volatility is the most important proxy of arbitrage costs, as it is correlated with transaction costs and imposes a significant holding cost for arbitrageurs. Thus, financial intermediaries would charge extra compensation for supplying these options, which leads to higher prices and lower returns.

The return of the equity options has been a popular topic in the literature. Hu and Jacobs (2014) provide a theoretical and empirical analysis of the relationship between expected option returns and the volatility of the underlying assets. They find the raw call option return is a decreasing function of the volatility of the underlying assets, while the raw put option return is increasing with the volatility of the underlying assets. Aramonte (2014) finds that macroeconomic uncertainty is priced in the cross-section of option returns, even after controlling for a number of relevant factors.

One of the crucial issues in empirical option pricing is model specification. Christoffersen, Fournier, and Jacobs (2013) find that the principal component analysis of equity options on Dow-Jones firms reveals a strong factor structure. They further develop an equity option valuation model that captures the cross-sectional market factor structure as well as stochastic volatility through time. The model assumes a Heston (1993) style stochastic volatility model for the market return but additionally allows for stochastic idiosyncratic volatility for each firm; thus, it is referred as β-Heston model.

In this paper, I follow the methodology from Broadie, Chernov, and Johannes (2009) to investigate the cross-sectional returns of 29 individual equity options (from Dow Jones Industrial Average index (DJIA)). The β-Heston model is used as a benchmark to assess the evidence for equity option mispricing. Option returns computed from formal option pricing models automatically reflect the leverage and kinked payoffs of options, and anchor hypothesis tests at null values, provide a framework for assessing statistical uncertainty via simulations. Furthermore, option returns are more straightforward to interpret economically than pricing errors. Returns represent actual gain or losses on purchased securities.

First, I find that, compared to index option returns, individual equity option returns are highly volatile; their patterns are less clear and not easily traceable. Thus, comparing each of the equity option average returns to those generated by the model found few interesting results. However, if we take these 29 equity options' average returns as a whole, and then compare this distribution to that generated by the β-Heston model, I find that the two distributions do not differ significantly from each other, indicating that the β-Heston model could provide key insights for understanding and evaluating equity put returns. The overall performance of equity options is consistent with model estimation.

Meanwhile, the hypothesis that skewness preference influences asset prices is also tested in this paper. Recent studies have found that total skewness is priced in stocks. However, this literature also concludes that estimating ex ante skewness for option returns is quite difficult because the correct set of predictive instruments is not known. Boyer and Vorkink (2014) introduce an ex ante option return skewness measurement that is simple to construct. It only relies on three variables: moneyness, underlying asset expected return and volatility. Compared to previous studies, I modify their methods to compute the ex ante skewness in order to exploit the information embedded in the model. First, I find that this new parametric expected measurement is able to replicate the results from Boyer and Vorkink (2014). There is a negative and robust relationship between expected skewness and equity option returns. The spread between the Low Skewness portfolio and the High Skewness portfolio is positive and significant across all maturities. Again, I apply the simulation procedure to test the null hypothesis that skewness is not priced. However, the simulations under the β-Heston model produce very similar patterns as the actual data; indeed, they are statistically insignificant when compared to each other. Consequently, we cannot reject the null hypothesis that skewness is not priced in the cross-section of individual equity option returns.

Furthermore, I also provide evidence that the negative relationship between delta-hedged equity option returns and idiosyncratic volatility of the underlying stocks can be replicated by model simulations, which casts doubt on the hypothesis of market imperfections and constrained financial intermediaries.

The remainder of this paper is structured as follows. Section 2 introduces the β-Heston Model. Section 3 discuss the dataset and estimation methods. Section 4 investigates the mispricing issue based on the β-Heston model. Section 5 shows how to construct skewness portfolios and compares the actual returns with artificial returns generated from the model. Section 6 provides evidence that casts doubt on the hypothesis of market imperfections and constrained financial intermediaries. Conclusions are given in Section 7.

2 Model

In the option pricing literature, it is typical to assume a stochastic process for each underlying equity price. Option pricing based on this stochastic process ignores any links the underlying equity prices may have with other equity prices through common factors. When considering a single stock option, ignoring an underlying equity factor structure maybe be harmless. However, it is crucial in portfolio management to understand links between the underlying stocks or options.

2.1 Physical Measure

In this paper, following Christoffersen et al. (2013), I consider an equity market consists of n firms driven by a single market factor, I_t (index). The individual stock prices are denoted by S_t^j, for j=1,2,...,n. The market factor has the following dynamics:

$$\frac{dI_t}{I_t} = (r + \mu_I)dt + \sigma_{I,t} dW_t^{I,1} \tag{1}$$

$$d\sigma_{I,t}^2 = \kappa_v(\theta_v - \sigma_{I,t}^2)dt + \delta_I \sigma_{I,t} dW_t^{I,2} \tag{2}$$

where μ_I is the instantaneous market risk premium, θ_I denotes the long-run variance, κ_I captures the speed of mean reversion of $\sigma_{I,t}^2$ to θ_I, and δ_I measures volatility of volatility. The innovations to the market factor return and volatility are correlated with coefficient ρ_I.

Individual equity prices are driven by the market factor as well as an idiosyncratic term which also has stochastic volatility:

$$\frac{dS_t^j}{S_t^j} - rdt = \alpha_j dt + \beta_t^j\left(\frac{dI_t}{I_t} - rdt\right) + \sigma_{j,t} dW_t^{j,1} \tag{3}$$

$$d\sigma_{j,t}^2 = \kappa_j(\theta_j - \sigma_{j,t}^2)dt + \delta_j \sigma_{j,t} dW_t^{j,2} \tag{4}$$

where α_j denotes the excess return (equity premium) and β_j is the market beta of firm j.

2.2 Risk-Neutral Measure

According to the equivalent martingale measure, the $\mathbb{Q}$-process of the market factor is given by:

$$\frac{dI_t}{I_t} = rdt + \sigma_{I,t} dW_t^{\mathbb{Q}(I,1)} \tag{5}$$

$$d\sigma_{I,t}^2 = \kappa_v^{\mathbb{Q}}(\theta_v^{\mathbb{Q}} - \sigma_{I,t}^2)dt + \delta_I \sigma_{I,t} dW_t^{\mathbb{Q}(I,2)} \tag{6}$$

and the $\mathbb{Q}$-processes of the individual equities are given by:

$$\frac{dS_t^j}{S_t^j} = rdt + \beta_t^j(\frac{dI_t}{I_t} - rdt) + \sigma_{j,t} dW_t^{\mathbb{Q}(j,1)} \tag{7}$$

$$d\sigma_{j,t}^2 = \kappa_j(\theta_j - \sigma_{j,t}^2)dt + \delta_j \sigma_{j,t} dW_t^{\mathbb{Q}(j,2)} \tag{8}$$

Note that the market factor structure is preserved under $\mathbb{Q}$. The market beta is the same under both risk-neutral and physical measure. This is consistent with Serban, Lehoczky, and Seppi (2008), who document that the risk-neutral and objective betas are economically and statistically close for most stocks.

It should also be noticed that κ_j and θ_j are the same under both $\mathbb{P}$ and $\mathbb{Q}$ measure, indicating that the idiosyncratic variance risk is not priced. Put it differently, all of the risk premium (except for the equity premium) which is defined as the difference between physical and risk neutral measure is explained by the market factor through β.

2.3 Closed-Form Option Price

The model discussed before is affine. It implies that the characteristic function for the log equity price can be derived analytically. The characteristic function for the market index will be exactly identical to that in Heston (1993). While for the individual equity options, the risk-neutral conditional characteristic function $\phi_{t,T}^{\mathbb{Q},j}(u)$ is given by

$$\phi_{t,T}^{\mathbb{Q},j}(u) = (S_t^j)^{iu} exp(iur(T-t) - A^I(\Lambda^S, u) - B^I(\Lambda^S, u)\sigma_{I,t}^2 - A^j(\Lambda^S, u) - B^j(\Lambda^S, u)\sigma_{j,t}^2). \tag{9}$$

The expression for A^I, B^I, A^j, and B^j can be found in the appendix. Given the spot price characteristic function under $\mathbb{Q}$, the price of a European equity call option with strike price K and maturity $T-t$ is

$$C_t^j(K, T-t) = S_t^j \Pi_1^j - Ke^{-r(T-t)}\Pi_2^j \tag{10}$$

where the risk-neutral probabilities Π_1^j and Π_2^j are defined by:

$$\Pi_1^j = \frac{1}{2} + \frac{e^{-r(T-t)}}{\pi S_t^j} \int_0^\infty Re\Big[\frac{e^{-iulnK}\phi_{t,T}^{\mathbb{Q}}(u-i)}{iu}\Big] du \tag{11}$$

$$\Pi_2^j = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty Re\Big[\frac{e^{-iulnK}\phi_{t,T}^{\mathbb{Q}}(u)}{iu}\Big] du \tag{12}$$

with conditional characteristic function $\phi_{t,T}^{\mathbb{Q}}$ defined before.

3 Estimation Methods

Several methods have been proposed in the literature for estimating stochastic volatility model with latent variables, including MCMC, EMM, IS-GMM

(Pan (2002) proposed Implied-State GMM estimation strategy. The author approximates the unobserved volatility, V_t, with an option-implied volatility which is inverted from the time-t spot price and a near-the-money short-dated option price) and so on. Another approach treats the latent variable as a parameter to be estimated and thus avoids filtering problem.

3.1 Data

I collect historical daily data on S&P500 and 29 equity options from January 1996 to August 2014. S&P500 index are used to proxy for the market factor. The individual equities are selected from the Dow Jones Industrial Average index (DJIA). Of the 30 firms in the index, Kraft Foods is excluded for which OptionMetrics only has data from 2001. I filter out options having more than 365 days to maturity. Following Bakshi, Cao and Chen (1997), I use mid-quotes (average bid-ask spread) in all computations, and eliminate options with moneyness (K/S) less than 0.9 or greater than 1.1. I also filter out quotes with implied Black-Scholes Vega equal to zero. The S&P500 index options are European, but the individual equity options are American style. As a result, their prices are influence by early exercise premium. To circumvent possible biases due to the presence of early exercise premium, I eliminate in-the-money (ITM) options for which the early exercise premium matters most (See also Bakshi, Kapadia, and Madan (2003)).

3.2 Volatility Series

In this model, two vectors of latent variables $\{\sigma^2_{I,t}, \sigma^2_{I,t}\}$ and two sets of structural parameters $\{\Theta_I, \Theta_j\}$ need to be estimated where $\Theta_I \equiv \{\kappa_I^{\mathbb{Q}}, \theta_I^{\mathbb{Q}}, \delta_I, \rho_I\}$, and $\Theta_j \equiv \{\kappa_j, \theta_j, \delta_j, \rho_j \, \beta_j\}$. The parameters Θ_I and Θ_j are taken from Christoffersen et al. (2013), the details of estimation procedure can be found in the appendix. While the volatility series of market index and equities are estimated using the least square method.

$$\hat{\sigma}^2_{I,t} = arg \min_{\sigma^2_{I,t}} \sum_{m=1}^{N_{I,t}} (C_{I,t,m} - C_m(\Theta_I, \sigma^2_{I,t}))^2 / Vega^2_{I,t,m}, t = 1, 2, ...T \tag{13}$$

Where $C_{I,t,m}$ is the market price of index option contract m quoted at t, $C_m(\Theta_I, \sigma^2_{I,t})$ is the model index option price, $N_{I,t}$ is the number of index contracts available on day t, and $Vega_{I,t,m}$ is the Black-Scholes sensitivity of the option price with respect to volatility evaluated at the implied volatility. These vega-weighted price errors are good approximation to implied volatility errors and they are much more quickly computed. This method has been used in Carr and Wu (2007).

Given an initial value Θ_j and using the estimated $\hat{\sigma}^2_{I,t}$ and $\hat{\Theta}_I$, we can estimate the spot equity variance each day by sequentially solving

$$\hat{\sigma}^2_{j,t} = arg \min_{\sigma^2_{j,t}} \sum_{m=1}^{N_{j,t}} (C_{j,t,m} - C_m(\Theta^0_j, \sigma^2_{j,t}))^2 / Vega^2_{j,t,m}, t = 1, 2, ...T \tag{14}$$

3.3 GMM

Since I treat the latent variable as parameters, indeed $\sigma_{I,t}$ and $\sigma_{j,t}$ are estimated in the previous step, then it is natural to use standard GMM method to estimate $\mathbb{P}$-parameters. Given the characteristic function solved before, we can write conditional moment generating function based on the following relationship:

$$M_X(t) = \phi(-it) \tag{15}$$

Letting

$$y_n = lnS_n - lnS_{n-1} \tag{16}$$

denote the date-n return. We can construct n moment conditions by

$$G_N(\theta) = \frac{1}{N} \sum_{n \leq N} h\big(y_n, \sigma_n^2, \theta\big), \tag{17}$$

h is some test function be to chosen, such that

$$E_{n-1}^{\theta_0}\big[h(y_n), \sigma_n, \theta_0\big] = 0 \tag{18}$$

where θ_0 is the true model parameters, E_{t-1}^{θ} denotes the conditional expectation associated with parameter set θ. Define the GMM estimator $\hat{\theta}_N$ by

$$\hat{\theta}_N = arg \min_{\theta \in \Theta} G_N(\theta)^\top W_N G_N(\theta). \tag{19}$$

Given the explicitly known moment-generating function $M_X(t)$ defined before, the conditional moments of the log returns (setting $S_t = 1$ in ϕ) can be derived by

$$E_n(y_{n+1}^i) = \left.\frac{\partial^i M_X(u)}{\partial^i u}\right|_{u=0}, i \in \{0, 1, ...\}. \tag{20}$$

Let $m_i(\theta, \sigma_n^2) = E_n^{\theta}(y_{n+1}^i)$ and $h(y_n) = y_n^i - m_i(\theta, \sigma_n^2)$.

3.4 Parameters: Result

Broadie, Chernov and Johannes (2007) argued that the absolute continuity requirement implies that certain model parameters, are the same under both measures. In this model, a comparison of the dynamics of S_t under physical and risk-neutral measure demonstrate that $\kappa_j, \theta_j, \delta_j$ and ρ_j are the same under both measures. This implies that these parameters can be estimated either by index/equity returns or option prices, however, the estimates should be the same from either data source. As advocated by Bates (2000), in order to impose this theoretical restriction, we should constrain these parameters to be equal under both measures. For the parameters that are theoretically constrained to be equal across measures, I use $\mathbb{Q}$-measure parameters estimated by Christoffersen, Fournier and Jacobs (2013). The spot volatility is estimated in order to perform the standard GMM method. The results are summarized in Table 1.

[Insert Table 1 about here]

4 Expected option return

In light of Broadie et al. (2009), hold-to-expiration put returns are defined as

$$r^p_{t,T} = \frac{(K - S_{t+T})^+}{P_{t+T}(K, S_t)} - 1 \tag{21}$$

where $x^+ \equiv$ max(x,0) and $P_{t,T}(K, S_t)$ is the time-t price of a put option written on S_t, struck at K, and expiring at time t+T. Hold-to-expiration returns are widely analyzed in both academic studies and in practice given the fact that option trading involves significant cost, for example, ATM (OTM) index option bid-ask spreads are currently on the order of 3-5% (10%) of the option price. The goal of this section is to assess whether or not equity option returns are excessive, either in absolute terms or relative to their risks. It is common to compute average returns or Sharpe ratios to measure the performance of the asset returns. Strategies that writing put options generally deliver higher average returns than the underlying asset, have economically and statistically higher Sharpe ratios than the market.

It is well known that options are leveraged positions in the underlying asset, so call (put) options have higher (lower) expected returns than the underlying. The precise magnitude of expected returns depends on a number of factors that include the specific model, the parameters, and factor risk premiums. Previous studies concluded that EORs (expected option returns) are very sensitive to both the equity premium and volatility.

The experiment performed in this section is straightforward: I compare the observed values of these intuitive metrics (average returns and Sharpe ratios) in the data to those generated by the β-Heston model. The formal model provides an appropriate null value for anchoring hypothesis test.

4.1 Theoretical Predication Under the β-Heston Model

The β-Heston model provides theoretical background to study the equity option returns, more precisely, we are interesting in whether/how much the idiosyncratic return can be explained by the systematic risk factor. Christoffersen et al. (2013) provide an expression for the expected equity option returns as a function of the expected market return.

For a derivative f_j written on the stock price , S_t^j, the expected excess return on the derivative contract is given by:

$$\frac{1}{dt}E_p^{\mathbb{P}}[\frac{df^j}{f^j} - rdt] = \frac{\partial f^j}{\partial S_t^j}\frac{S_t^j}{f^j}\alpha_j + \frac{\partial f^j}{\partial I_t}\frac{I_t}{f^j}\mu_I = \frac{\partial f^j}{\partial S_t^j}\frac{S_t^j}{f^j}(\alpha_j + \beta_j\mu_I) \tag{22}$$

where $\frac{\partial f_j}{\partial I_t}$ is the sensitivity of derivative contract f^j with respect to the index level, I_t (the market delta). It is given by

$$\frac{\partial f^j}{\partial I_t} = \frac{\partial f^j}{\partial S_t^j}\frac{S_t^j}{I_t}\beta_j. \tag{23}$$

This result reveals that the beta of the stock provides a simple link between the expected return on the market index and the expected return on the equity option via the delta of the option. The model thus decomposes the excess return on the options into two parts: The delta of the equity option and the beta of the stock. In other words, equity options provide investors with two sources of leverage: first, the beta with respect to the market, and second, the elasticity of the option prices with respect to changes in the stock price.

[**Insert Figure 1 about here**]

In Figure 1, I plot the expected hold-to-expiration returns on equity call options (top panel) and on put options (bottom panel) in percent per month against moneyness for firms with different betas. The top panel of Figure 1 shows that the difference in expected call returns across firms with different betas can be substantial for OTM calls where option leverage in general is high. The bottom panel of Figure 1 shows that put option expected returns (which are always negative) also vary across firms with different betas, when the put options are OTM. As the formula implied, the betas play different roles in expected return for call and put options since the delta of call and put options is opposite. For call options, given a moneyness group, higher beta indicates higher return. While for put options, the relationship is reversed.

4.2 Analytical expected option returns

Expected put option returns are given by

$$E_t^{\mathbb{P}}(r_{t,T}^p) = \frac{E_t^{\mathbb{P}}[(K - S_{t+T})^+]}{P_{t+T}(S_t, K)} - 1 = \frac{E_t^{\mathbb{P}}[(K - S_{t+T})^+]}{E_t^{\mathbb{Q}}[e^{-rT}(K - S_{t+T})^+]} - 1 \qquad (24)$$

It is clear that any model that admits analytical option prices, such as affine models, will allow EORs to be computed explicitly since the numerator and denominator are known analytically. EORs do not depend on S_t. To see this, define the initial moneyness of the option as $\kappa = K/S_t$. Option homogeneity implies that

$$E_t^{\mathbb{P}}(r_{t,T}^p) = \frac{E_t^{\mathbb{P}}[(\kappa - R_{t+T})^+]}{E_t^{\mathbb{Q}}[e^{-rT}(\kappa - R_{t+T})^+]} - 1, \tag{25}$$

where $R_{t,T} = S_{t+T}/S_t$ is the gross index return. EORs depend only on the moneyness, maturity, interest rate, and distribution of the underlying returns.

These analytical results are primarily useful as they allow us to assess the exact quantitative impact of risk premiums or parameter configurations. Equation (32) implies the gap between the $\mathbb{P}$ and $\mathbb{Q}$ probability measures determines EORs, and the magnitude of the returns is determined by the relative shape and location of the two probability measures. In models without jump or stochastic volatility risk premiums, the gap is determined by the equity risk premium. When we take stochastic volatility or jump risk premium into consideration, both the shape and location of the distribution can change, leading to more interesting patterns of expected returns across different moneyness categories.

4.3 Summary for Equity Option Returns

Options analyzed in this section are one month time-to-maturity OTM put options. Hold-to-expiration returns are computed for fixed moneyness, measured by strike divided by the underlying (K/S_t), ranging from 0.92 to 1.00 (in 2% increments).

Table 2 shows the average hold-to-expiration returns for 29 equity options divided into five moneyness groups. As we can see from the table, the equity option returns are highly volatile, for moneyness equal to 0.92, returns range from -88% for JNJ (Johnson & Johnson) to 35% for TRV (Travellers) per month. The mean returns for each moneyness group are negative and increasing with moneyness. These patterns are consistent with the prediction derived in Coval and Shumway (2001) under general assumptions.

[Insert Table 2 about here]

We can find similar results from Table 3, the distribution of Sharpe ratios for each moneyness group is highly volatile. Generally, the Sharpe ratios of put options are larger (in absolute value) than those of the underlying market. For instance, the monthly Sharpe ratio for the market is about 0.1, and the put return Sharpe ratios are several times larger.

[Insert Table 3 about here]

4.4 Return Distribution via Monte Carlo simulation

To assess statistical significance, I use Monte Carlo simulation to compute the distribution of various returns statistics, including average returns and Sharpe ratios. I simulate $N = 10000$ times of index and 29 equities levels using Milstein scheme simulation. For each equity j and underlying simulation trial g, put returns for a fixed moneyness κ are

$$r_{t,T}^{j,\kappa,(g)} = \frac{(\kappa - R_{t,T}^{(g)})^+}{P_T(\kappa)} - 1 \tag{26}$$

where

$$P_T(\kappa) \equiv \frac{P_{t,T}(S_t, K)}{S_t} = e^{-rT} E_t^{\mathbb{Q}}[(\kappa - R_{t,T})^+] \tag{27}$$

$g = 1, ..., N$. Average simulated returns for the equity option j, from moneyness group κ are

$$\bar{r}_{t,T}^{j,\kappa} = \frac{1}{N} \sum_{g=1}^{N} r_{t,T}^{j,\kappa,(g)} \tag{28}$$

Similarly, we can construct finite-sample distributions for Sharpe ratios. The following subsection illustrates the simulation techniques.

4.5 Results From Simulations

Table 4 summarizes EORs (expected option returns) corresponding to each equities for various strikes. It is assumed that all risk premiums (except for the equity premium) are equal to zero. The simulated returns are relatively stable, compared to those generated by real data. For each of the equities, EORs are increasing with respect to moneyness. This pattern becomes clear in the simulated returns.

[**Insert Table 4 about here**]

As we already seen, the equity option returns are volatile and their patterns are less clear. Thus, comparing each of the equity option average returns to those generated by the model could hardly find any interesting results. However, if we take these 29 equities options as a whole, which means their average returns constitute a distribution of individual equity option average returns. Then compare this distribution to the one generated by the β-Heston model, I find that the p-value is quite high, indicating that two distributions are not significantly different from each other. Similar result is also found for Sharpe Ratios in Table 5. The results are summarized in Table 6.

[**Insert Table 5 about here**]

The top panel of Table 6 reports population average returns for put options of 29 equities for various strikes. We first note that all the metrics except 8% OTM option return are statistically insignificant when compared to the model. Based on the β-Heston model, we can conclude that: generally, the β-Heston model could provide key insights for understanding and evaluating equity put returns. This result is interesting since the existing literature concludes that OTM put options are most anomalous or mispriced. The results for Sharpe ratios are similar, with none of the strikes statistically different from those generated by the model. These two findings indicate that the overall performance of the equity options is consistent with model estimation. This result is particularly useful when we are evaluating the performance of portfolios consist of equity options. It is proved indirectly in the next section.

[**Insert Table 6 about here**]

5 Skewness Preference and Option Returns

Recent research shows that individuals deviate from standard utility theory when making choices in the face of uncertainty. For instance, investors prefer skewness or lottery-like features in asset return distributions, and these preferences influence asset prices in equilibrium. Asset returns have a strong negative relationship with skewness. The individual equity options market offers an ideal platform to study the skewness preference on asset returns. The unusual dramatic lottery-like features in option returns due to the implicit leverage in an option contract combined with a nonlinear payoff. Empirically, the ex ante return skewness of equity options can be more than 10 times higher than equity return skewness. Previous studies suggest that total skewness is priced. Boyer and Vorkink (2014) find a significant and economically large effect of total skewness preference on option prices in both call and put option markets.

5.1 Ex ante skewness measurement

To understand whether differences in the lottery-like characteristics of options help explain cross-sectional variation in their expected returns, it is assumed that skewness is a proxy for the lottery-like characteristics of options. In light of Boyer and Vorkink (2014), I construct closed-form ex ante skewness measures for the physical distribution of option returns by integrating the appropriate PDF under the assumption that stock returns are lognormal.

It is obvious that the lognormal assumption does not perfectly characterize the distribution of the underlying stocks. However, it allows for a simple approach to estimate the physical ex ante skewness of an option contract that uses only information available at the time of purchase. Lien (1985) provided a closed-form moments for options returns (under this assumption) by integrating the truncated lognormal PDF.

The ex ante skewness for option j over hirizon t to T is defined as

$$sk_{j,t:T} = \frac{E_t[R_{j,t:T} - \mu_{j,t:T}]^3}{[\sigma_{j,t:T}]^3} \tag{29}$$

where $R_{j,t:T}$ denotes option j's hold-to-expiration return defined before, $E_t[.]$ denotes the expectation given information known at time t, $\mu_{j,t:T} = E_t[R_{j,t:T}]$, and $\sigma_{j,t:T} = (E_t[R^2_{j,t:T}] - \mu^2_{j,t:T})^{1/2}$. By rewriting previous equation in terms of its raw moments,

$$sk_{j,t:T} = \frac{E_t[R^3_{j,t:T}] - 3E_t[R^2_{i,t:T}]\mu_{i,t:T} + 2\mu^3_{i,t:T}}{[E_t[R^2_{i,t:T}] - \mu^2_{i,t:T}]^{1.5}}, \tag{30}$$

note that only the first three raw moments of the option return are required to calculate $sk_{i,t:T}$. Given the definition of hold-to-expiration return, we can write the m^{th} raw moment for put option j as

$$E_t\big[(R^p_{j,t:T})^m\big] = E_t\left[(\frac{K_j - S_{j,T}}{P_{j,t}})^m | K_j > S_{j,T}\right] P_t(K_j > S_{j,T}) \tag{31}$$

where $P_t(.)$ indicates the probability given information at time t. Under the assumption of lognormality, equation (42) illustrates the raw moments for a put option are a function of the raw moments of a truncated lognormal distribution. The following section demonstrates how to construct the expected skewness measure, $sk_{i,t:T}$.

5.2 Closed form of raw raw moments

Let $r = ln(S_T/S_t)$, the log stock return, and assume that r is distributed $N(\mu, \sigma^2)$. Under this assumption, the stock return, S_T/S_t, is lognormal. In the original paper of Boyer and Vorkink (2014), μ_j and σ^2_j are estimated using six months of daily data prior to t. While in this paper, I estimate these two variables in a parametric way in order to absorb the firm specific information contained in the model. According to the β-Heston model, for equity j with options expire at T,

$$\mu_j = (r + \alpha_j + \beta_j \mu_I) \times (T - t) \tag{32}$$

and

$$\sigma_j^2 = \big(E_t\big[V_{j,T}\big] + \beta_j^2 E_t\big[V_{I,T}\big]\big) \times (T-t), \tag{33}$$

with

$$E_t\big[V_{i,T}\big] = e^{-\kappa_i(T-t)}V_{i,t} + \theta_i(1-e^{-\kappa_i(T-t)}) \tag{34}$$

where $i = I, j$ stands for the index process and equity process, respectively. $\alpha_j, \beta_j, \mu_I, \kappa_i$, and θ_i are the parameters estimated before. $V_{i,t}$ is set equal to average spot variance of the underlying.

It should be emphasized that estimating μ and σ^2 in this parametric way does not rely on the distribution of the β-Heston model, instead, it provides an approximation for these two variables. Given the raw moments for put options, we can construct $sk_{j,t:T}$ for put options for any level of moneyness and maturity.

5.3 Option Characteristics and Skewness

In order to understand how different option characteristics could influence the expected skewness measure, $sk_{i,t:T}$, Figure 2 plots $sk_{i,t:T}$ as a function of moneyness K/S_t, for three different time to maturities. Although the way to estimate $sk_{i,t:T}$ is slightly different from the one used in Boyer and Vorkink (2014), we both find that there is a relationship between moneyness and ex ante skewness, especially for short maturity options. Generally, out-of-the-money options offer higher skewness than in-the-money options. For instance, the ex ante skewness of short-term, out-of-the-money options is well over 10, several times large than the ex ante skewness of equity returns (See Boyer, Mitton, and Vorkink (2010) and Conrad, Dittmar, and Ghysels (2013)). Comparing top panel and bottom panel, we can find that call options display similar patterns as their corresponding put options. Figure 3 plots $sk_{i,t:T}$ as a function of moneyness K/S_t, for three different betas. We can see that Beta plays the same role for both call options and put options: All things equal, as the beta increase, the ex ante skewness decrease for both call and put options.

[**Insert Figure 2 about here**]

[**Insert Figure 3 about here**]

5.4 Option Portfolio Formation and Returns

Based on the parameters and equations given in the previous sections, we can now compute the ex ante skewness for each of the equity options. Table 7 illustrate the distribution of put option ex ante skewness for fixed moneyness with different maturities. Note that each of the cell (skewness value) in the table is attached to one real return and one simulated return corresponding to its parameters. Similar procedures are repeated for each of the five moneyness groups (OTM 8% to ATM). Next, for each portfolio maturity, I sort options within each expiration bin into ex ante skewness quintiles.

[**Insert Table 7 about here**]

Table 8 reports average of portfolio returns generated by data and simulations for each ex ante skewness/maturity bin. Both actual returns and simulated returns decrease dramatically across skewness bins for every maturity group. We first focus on the return generated by data. For example, put options that expire in two weeks, the actual average hold-to-maturity return is monotonically decreasing from -17% for the low skewness bin to -54% for the high skewness bin. The paired t-statistic for the difference is

7.738. Furthermore, the average difference in spreads between the low and high skewness portfolios is positive and significant in all cases. These results (real data) are similar to those reported by Boyer and Vorkink (2014). Based on these results (and some control tests), they claimed that skewness preference is priced in the equilibrium. Put differently, these results indicate that individual equity option investors give up average returns on the order of 50% monthly for exposure to the lottery opportunities that options with high ex ante skewness offer.

[**Insert Table 8 about here**]

However, the results from simulations provide evidence for an opposite conclusion. The simulated portfolio returns exhibit very similar patterns as actual returns (the monotonicity feature is even more clear). Again, the difference in spreads between the low and high skewness portfolios is positive and significant across all maturity groups, with substantially higher t-statistics. The p-values between simulated and real portfolio returns are generally high for each ex ante skewness/maturity bin. It indicates that the two distributions are not statistically different from each other. Consequently, we can not reject the null that skewness is not priced in the cross-section of individual equity option returns.

6 Stock Volatility and Option Returns

Cao and Han (2012) presents a robust finding that delta-hedged equity option return decreases monotonically with an increase in the idiosyncratic volatility of the underlying stock. The result is still significant even after controlling for standard risk factors. The intuition behind this finding could be market imperfections and constrained financial intermediaries: Dealers charge a higher premium for options with high idiosyncratic volatility underlying due to their higher arbitrage costs. This hypothesis is motivated by theory of option pricing in imperfect market that emphasizes the role of constrained financial intermediaries.

Option prices are affected by demand and supply from the markets when there are limits to arbitrage and it is costly to hedge or replicate the options. Shleifer and Vishny (1997) argue that the idiosyncratic volatility is the most important proxy of arbitrage costs, as it is correlated with transaction costs and imposes a significant holding cost for arbitrageurs. On the one hand, options with high idiosyncratic volatility attract high demand from speculators. On the other hand, such options are more difficult to hedge. Thus, financial intermediaries would charge extra compensation for supplying these options, which leads to a higher price and lower return. Hu and Jacobs (2014) provide a theoretical and empirical analysis of the relationship between expected option returns and the volatility of the underlying. They find the raw call option return is a decreasing function of the volatility of the underlying, while the raw put option return is increasing with the volatility of the underlying.

6.1 Delta-hedged option returns

I study the delta-hedged put option returns in this paper, using the Black-Scholes delta as approximation. Following Goyal and Saretto (2009), the strategy return is defined as hold-to-expiration, the position of stocks and options is fixed after the strategy is built. The details follows: For each of the month during the sample period, I long one unit of at-the-money put option with maturity equal to one month (if available). Then, hedge the put with a long position of Δ (Black-Scholes delta) unit of underlying stock. The one month hold-to-expiration return Γ of this strategy is defined as

$$\Gamma(t,T) = \frac{max(0, K - S_T) + \Delta_t \cdot S_T}{P_t + \Delta_t \cdot S_t} - 1 \tag{35}$$

Where t is the time when we build the strategy, P_t is the put price at that time, Δ_t is the Black-Scholes delta of put option at time t.

6.2 Underlying volatility and portfolio construction

The total volatility (VOL_{tot}) of the underlying is computed based on the daily log-return of the underlying price over the previous month, then annualized. Similarly, the market volatility (VOL_{mkt}) is computed based on the daily log-return of the S&P500 index over the previous month, then annualized. The idiosyncratic volatility is defined as follows:

$$VOL_{idio} = \sqrt{VOL_{tot}^2 - \beta_j^2 \cdot VOL_{mkt}^2} \tag{36}$$

Where β_j is the parameter estimated before for stock j.

The portfolio is constructed by sorting the underlying total/idiosyncratic volatility. At the maturity of the put option, I rank the strategy returns into five quintiles based on the underlying idiosyncratic volatility (same procedures are repeated for total volatility). Note that the simulated portfolio returns are sorting based on total long-term volatility $\sqrt{\theta_j + \beta_j^2 \cdot \theta_i}$ and idiosyncratic long-term volatility $\sqrt{\theta_j}$.

6.3 Results summary

Table 9 summarize the delta-hedged portfolio returns from actual data and simulations based on two sorting criteria: Total volatility and Idiosyncratic volatility. We first look at the actual data, it shows that the returns of delta-hedged put options are always negative (long position), furthermore, the average return on high total/idiosyncratic volatility stocks is significantly higher than that on low total/idiosyncratic volatility stocks. For instance, the average difference in returns between the portfolio of long positions in delta-hedged puts for stocks ranked in the top volatility quintile and that for stocks ranked in the bottom volatility quintile is 1.6%, with a $t-statistic$ equal to 5.14. Similar result (1.05 % with $t-statistic$ equal to 2.96) can be found when we sort stocks by their idiosyncratic volatility. Although in Cao and Han (2012), they use daily-hedge strategy (while in this paper, the position is hedged and fixed at the initiative), the negative relationship between portfolio returns and underlying volatility is also confirmed here.

[**Insert Table 9 about here**]

However, this pattern can be also found qualitatively in the corresponding simulation portfolios. As we can see from the table, the difference in simulated returns between high volatility quintile and low volatility quintile is 0.43% for sorting total volatility and 0.31% for sorting idiosyncratic volatility. The difference is statistically significant, however, it is much smaller than the one from actual data. This is due to the fact that the simulated average returns are more flat across different quintiles, compare to those from real data. The mean of the average returns from different quintiles is almost the same for both Data portfolios and Simulated portfolios, with a $P-value$ equal to 0.71 and 0.68 for total and idiosyncratic volatility portfolio, respectively. As we can expected, when comparing data with simulations (measured by $P-value$), the portfolios from top/bottom quintile are significantly different from each other. While it is not the case in the middle quintiles, indeed, we can not reject the null that the average return is the same for Data and

Simulation portfolio for quintile 2, 3 and 4.

Although the results from the actual data illustrate that there are more extreme returns in the top/bottom quintile, the simulations still provide qualitatively similar pattern: the negative relationship between delta-hedged return and underlying volatility, cast doubts on the market imperfection and constrained financial intermediaries hypothesis.

7 Conclusion

In this paper, I study the cross-section of equity option returns to investigate the out-of-the-money option mispricing issue. The newly developed β-Heston model is used to construct sample distributions of average option returns and Sharpe ratios using Monte Carlo simulation. First, I find that the most puzzling, the very large (in absolute value) returns to OTM options is consistent with the β-Heston model. Second, I find little added benefit from using Sharpe ratios as diagnostic tools since the result is similar to those from average option returns.

Recent studies show that standard rational asset pricing models have difficult explaining many of the basic empirical facts about the financial markets. For instance, investors prefer skewness or lottery-like features in asset return distributions, and these preferences influence asset prices in equilibrium. I modify the method to compute the ex ante skewness (in a parametric way) in order to exploit the information from the model. First, I find that this new parametric ex ante skewness measurement is able to replicate the results from Boyer and Vorkink (2014). There is a negative and robust relationship between ex ante skewness and equity option returns. Then, I apply the simulation procedure to test the null that skewness is not priced. However, different from previous studies, the simulation under the β-Heston model produces very similar patterns as the actual data, indeed, they are not statistically different from each other. Consequently, we can not reject the null that skewness is not priced in the cross-section of individual equity option returns.

Furthermore, I also provide evidence that the negative relationship between delta-hedged equity option returns and idiosyncratic volatility of the underlying stocks can be replicated by model simulations, which casts doubts on the hypothesis of market imperfections and constrained financial intermediaries.

Note that these findings should not be interpret as the evidence that the β-Heston model is correct, but rather as highlighting the statistical difficulties present when analyzing option returns. Indeed, a natural extension to the β-Heston model is to incorporate jumps in the underlying as well as volatility process. It would be interesting to study how would expected option returns change due to these innovations. I leave these questions for future work.

A Appendix A: Ex Ante Skewness

In the appendix, following Boyer and Vorkink (2014), I demonstrate how the ex ante skewness measure, $sk_{j,t:T}$ is constructed based on the assuming lognormal stock prices. In light of Lien's (1985) theorem regarding truncated lognormal distributions, theorem A.1 is presented here.

Theorem A.1: *Let* $(u_1, u_2)'$ *be a normal random vector with mean (0,0)' and covariance matrix=* $\begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix}$. *Then*

$$E(exp(ru_1 + su_2)|u_1 > a) = N(\frac{h-a}{\sigma_1})\frac{exp[-D/2Q]}{N(\frac{-a}{\sigma_1})}, \tag{37}$$

where $h = r\sigma_1^2 + s\sigma_{12}, D = -Q(r^2\sigma_1^2 + 2rs\sigma_{12} + s^2\sigma_2^2), Q = \sigma_2^2 sigma_1^2 - sigma_{12}^2, andN(\cdot)$ is the CDF of the normal.

Note first that Lien's (1985) theorem A.1 can be used to derive closed-form solutions for the raw moments of option returns given by equation (42). These raw moments can be substituted into equation (41) to construct $sk_{j,t:T}$. For $m = 1$, equation (42) can be written as

$$E[R^c_{t:T}] = [\frac{S_t}{C_t}E(\frac{S_T}{S_t}|\frac{S_T}{S_t} > \frac{X}{S_t}) - \frac{X}{C_t}]P(\frac{S_T}{S_t} > \frac{X}{S_t}), \tag{38}$$

where S_t is the value of the underlying asset at time $t < T$. Let $\hat{r} = ln(S_T/S_t)$, the log stock return, and define A as $A = lon(X/S_t)$. Then equation (52) can be written as

$$E[R^c_{t:T}] = [\frac{S_t}{C_t}E(e^{\hat{r}}|\hat{r} > A) - \frac{X}{C_t}]P(\hat{r} > A). \tag{39}$$

Now assume that $\hat{r}$ is distributed $N(\hat{\mu}, \hat{\sigma}^2)$, where in general $\hat{\mu}$ can be nonzero. Under this assumption, the stock return, S_T/S_t, is lognormal. Furthermore, define $z = \hat{r} - \hat{\mu}$, so that z is distributed $N(0, \hat{\sigma}^2)$. Then note that

$$E(e^{\hat{r}}|\hat{r} > A) = E(e^{z+\hat{\mu}}|z > A - \hat{\mu}) = e^{\hat{\mu}}E(e^{z}|z > A - \hat{\mu}) \tag{40}$$

Then applying Lien's (1985) theorem implies that equation (54) can be written as

$$E(e^{\hat{r}}|\hat{r} > A) = \frac{exp[\hat{\mu} + \frac{\hat{\sigma}^2}{2}]N(d_1)}{N(d_2)} \tag{41}$$

Then we can plug equation (55) in to equation (53) to get the first moment of the call option return, following the similar approach, the corresponding raw moments for put options are

$$E\big[R^p_{t:T}\big] = \frac{KN(-d_2) - S_t exp\big[\frac{\sigma^2}{2} + \mu\big]N(-d_1)}{P_t} \tag{42}$$

where $d_1 = \frac{\sigma^2 + ln(S_t/K) + \mu}{\sigma}$ and $d_2 = d_1 - \sigma$.

$$\begin{aligned} E\big[(R^p_{t:T})^2\big] = & \frac{K^2N(-d_2) - 2XS_t exp\big[\frac{\sigma^2}{2} + \mu\big]N(-d_1)}{P_t^2} \\ & + \frac{S_t^2 exp\big[2\sigma^2 + 2\mu\big]N(-d_3)}{P_t^2} \end{aligned} \tag{43}$$

with $d_3 = d_1 + \sigma$, and $d_4 = d_1 + 2\sigma$.

$$\begin{aligned} E\big[\big(R^p_{t:T}\big)^3\big] = & \frac{3KS_t^2 exp\big[2\sigma^2 + 2\mu\big]N(-d_3) - S_t^3 exp\big[\frac{9}{2}\sigma^2 + 3\mu\big]N(-d_4)}{P_t^3} \\ & + \frac{K^3N(d_2) - 3K^2S_t exp\big[\frac{\sigma^2}{2} + \mu\big]N(-d_1)}{P_t^3} \end{aligned} \tag{44}$$

where P_t is the put price at time t and K is the strike price.

B Appendix B: Closed-Form Option Price

Christoffersen et al. (2013) provide the closed-form option price for the β-Heston model, the proof of the following result can be found in their paper.

The risk-neutral conditional characteristic function $\phi_{t,T}^{\mathbb{Q},j}(u)$ is given by

$$\phi_{t,T}^{\mathbb{Q},j}(u) = (S_t^j)^{iu} exp(iur(T-t) - A^I(\Lambda^S, u) - B^I(\Lambda^S, u)\sigma_{I,t}^2 - A^j(\Lambda^S, u) - B^j(\Lambda^S, u)\sigma_{j,t}^2). \tag{45}$$

Where

$$\begin{aligned} A^i(\Lambda, u) = \frac{\kappa_i^{\mathbb{Q}}\theta_i^{\mathbb{Q}}}{\delta_i^2}\Big\{ & 2ln(1 - \frac{\Psi^i(\Lambda^S, u) - \kappa_i^C}{2\Psi^i(\Lambda^S, u)}(1 - e^{-\Psi^i(\Lambda^S, u)(T-t)}) \\ & +(\Psi^i(\Lambda^S, u) - \kappa_i^C)(T-t)\Big\} \end{aligned} \tag{46}$$

$$B^i(\Lambda^S, u) = \frac{2g_h(u)(1 - e^{-\Psi^i(\Lambda^S, u)(T-t)})}{2\Psi^i(\Lambda^S, u) - (\Psi^i(\Lambda^S, u) - \kappa_i^C)(1 - e^{-\Psi^i(\Lambda^S, u)(T-t)})} \tag{47}$$

with

$$\Psi^i(\Lambda^S, u) = \sqrt{(\kappa_i^C)^2 + 2\delta_i^2 g_i(u)} \tag{48}$$

$$g_1(u) = \frac{iu}{2}\beta_j^2(1 - iu) \ \ and \ \ g_2(u) = \frac{iu}{2}(1 - iu) \tag{49}$$

$$\kappa_I^C = \kappa_I^{\mathbb{Q}} - iu\rho_I\beta_j\delta_I, \ \ \theta_I^C = \frac{\kappa_I^{\mathbb{Q}}\theta_I^{\mathbb{Q}}}{\kappa_I^C}, \ \ \kappa_j^C = \kappa_j - iu\rho_j\delta_j, \ \ \theta_j^C = \frac{\kappa_j\theta_j}{\kappa_j^C} \tag{50}$$

Note $i = I, j$ for index and equity, respectively. $h = 1$ if $i = I$ and $h = 2$ if $i = j$.

C Appendix C: Estimation Procedure

In this model, two vectors of latent variables $\{\sigma_{I,t}^2, \sigma_{I,t}^2\}$ and two sets of structural parameters $\{\Theta_I, \Theta_j\}$ need to be estimated where $\Theta_I \equiv \{\kappa_I^{\mathbb{Q}}, \theta_I^{\mathbb{Q}}, \delta_I, \rho_I\}$, and $\Theta_j \equiv \{\kappa_j, \theta_j, \delta_j, \rho_j\, \beta_j\}$. This involves two main steps. In the first step, the market index dynamic $\{\Theta_I, \sigma_{I,t}^2\}$ is estimated based on index option prices alone. In the second step, I take the market index dynamic as given, then estimate the firm-specific dynamics $\{\Theta_j, \sigma_{j,t}^2\}$. This step-wise estimation procedure (while not fully efficient) enables us to estimate the model for equities while ensuring that the same index dynamic is imposed for each of the individual equities. Christoffersen (2013) confirmed that this estimating technique has good finite sample properties in a Monte Carlo study.

Step 1: Market Index Volatility and Parameter Estimation Given a set of starting values, Θ_I^0, for the index structural parameters, I first estimate the spot market variance each day by sequentially solving

$$\hat{\sigma}_{I,t}^2 = arg \min_{\sigma_{I,t}^2} \sum_{m=1}^{N_{I,t}} (C_{I,t,m} - C_m(\Theta_I^0, \sigma_{I,t}^2))^2 / Vega_{I,t,m}^2, t = 1, 2, ...T \tag{51}$$

where $C_{I,t,m}$ is the market price of index option contract m quoted at t, $C_m(\Theta_I, \sigma_{I,t}^2)$ is the model index option price, $N_{I,t}$ is the number of index contracts available on day t, and $Vega_{I,t,m}$ is the Black-Scholes sensitivity of the option price with respect to volatility evaluated at the implied volatility. These vega-weighted price errors are good approximation to implied volatility errors and they are much more quickly computed. This method has been used in Carr and Wu (2007).

Once the set of T market spot variances have be obtained, we can solve for the set of market parameters as follows

$$\hat{\Theta}_I = arg\min_{\Theta_I} \sum_{m,t}^{N_I} (C_{I,t,m} - C_m(\Theta_I, \hat{\sigma}^2_{I,t}))^2 / Vega^2_{I,t,m}. \tag{52}$$

Iteration is needed between (20) and (21) until the improvement in fit is negligible.

Step 2: Equity Volatility and Parameter Estimation Given an initial value Θ_j^0 and using the estimated $\hat{\sigma}^2_{I,t}$ and $\hat{\Theta}_I$, we can estimate the spot equity variance each day by sequentially solving

$$\hat{\sigma}^2_{j,t} = arg\min_{\sigma^2_{j,t}} \sum_{m=1}^{N_{j,t}} (C_{j,t,m} - C_m(\Theta_j^0, \sigma^2_{j,t}))^2 / Vega^2_{j,t,m}, t = 1, 2, ...T \tag{53}$$

Once the set of T individual equity spot variance have be obtained, the set of individual equity parameters can be estimated as follows

$$\hat{\Theta}_j = arg\min_{\Theta_j} \sum_{m,t}^{N_j} (C_{j,t,m} - C_m(\Theta_j, \hat{\sigma}^2_{j,t}))^2 / Vega^2_{j,t,m}. \tag{54}$$

D Appendix D: Simulation Methods

The Euler scheme and the Milstein discretization are widely used in model simulation. The Euler scheme is a first-order method, it is the most basic explicit method for numerical integration of ordinary differential equations (ODE). While the disadvantage of the Euler scheme is its slow convergence. In this paper, I choose the Milstein scheme, which is a second-order method.

The corresponding scheme of discrete time stepping for index I_t is

$$I(t_{i+1}) = I(t_i) + I(t_i)\mu_I \Delta t + I(t_i)\sigma_I(t_i)\sqrt{\Delta t} W_i + \frac{1}{2}\sigma_I^2(t_i) I^2(t_i)\Delta t(W_i^2 - 1) \tag{55}$$

$$\sigma_I^2(t_{i+1}) = \sigma_I^2(t_i) + \kappa_I(\theta_I - \sigma_I^2(t_i))\Delta t_i + \delta_I \sigma_I(t_i)\sqrt{\Delta t} Z_i + \frac{1}{4}\delta_I^2 \Delta t(Z_i^2 - 1) \tag{56}$$

where W_i and Z_i are samples from a standard normal distribution with correlation equal to ρ_I. Note that $\mu_I, \kappa_I, \theta_I, \rho_I$ and δ_I are the parameters for the index process defined before.

The corresponding scheme of discrete time stepping for equity j is

$$\begin{aligned} S_j(t_{i+1}) = S_j(t_i) + S_j(t_i)(\alpha_j + r)\Delta t + \beta_j(\frac{I(t_{i+1}) - I(t_i)}{I(t_i)} - r\Delta t) \\ + S_j(t_i)\sigma_j(t_i)\sqrt{\Delta t} W_j + \frac{1}{2}\sigma_j^2(t_i) S_j^2(t_i)\Delta t(W_j^2 - 1) \end{aligned} \tag{57}$$

$$\sigma_j^2(t_{i+1}) = \sigma_j^2(t_i) + \kappa_j(\theta_j - \sigma_j^2(t_i))\Delta t_i + \delta_j \sigma_j(t_i)\sqrt{\Delta t} Z_j + \frac{1}{4}\delta_j^2 \Delta t(Z_j^2 - 1) \tag{58}$$

where W_j and Z_j are samples from a standard normal distribution with correlation equal to ρ_j. Note that $\alpha_j, \kappa_j, \theta_j, \beta_j, \rho_j$ and δ_j are the parameters for the equity process defined before.

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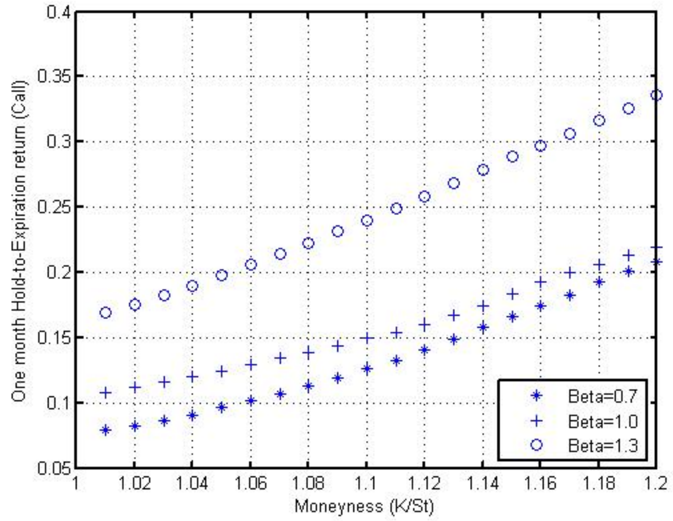
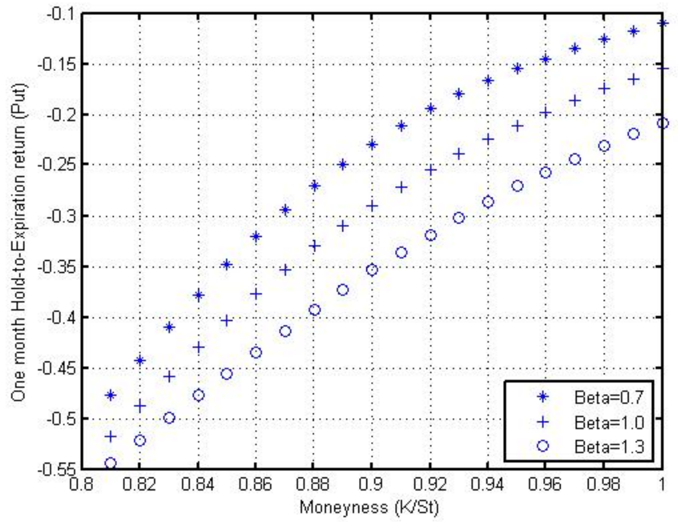
Figure 1: Expected Returns on Equity Options

Panel A: Call Options



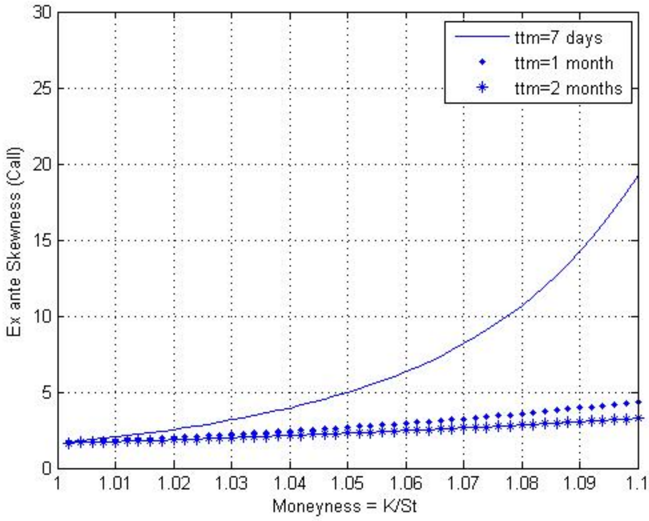
Panel B: Put Options



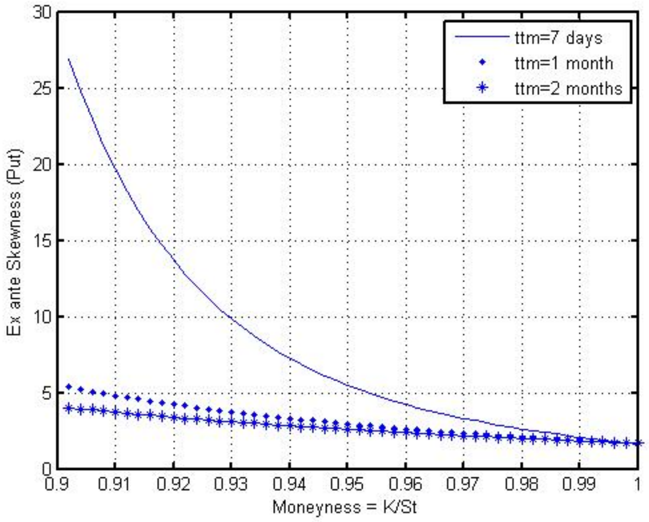
Note to Figure: In this figure, I plot the expected Hold-to-Expiration returns against betas on call and put using the model. Each line has a different beta. The parameters for the market index are $\kappa_I = 3.81$, $\theta_I = 0.0279$, $\delta_I = 0.456$, $\mu_I = 0.08$ and $\rho_I = -0.715$. The parameters for individual equity are $\kappa_j = 1.14$, $\theta_j = 0.0072$, $\delta_j = 0.128$, $\mu_j = 0.01$ and $\rho_j = -0.656$. The risk-free rate is set to 0.03. All the parameters are in annual basis.

Figure 2: Option skewness against moneyness

Panel A. Call Options



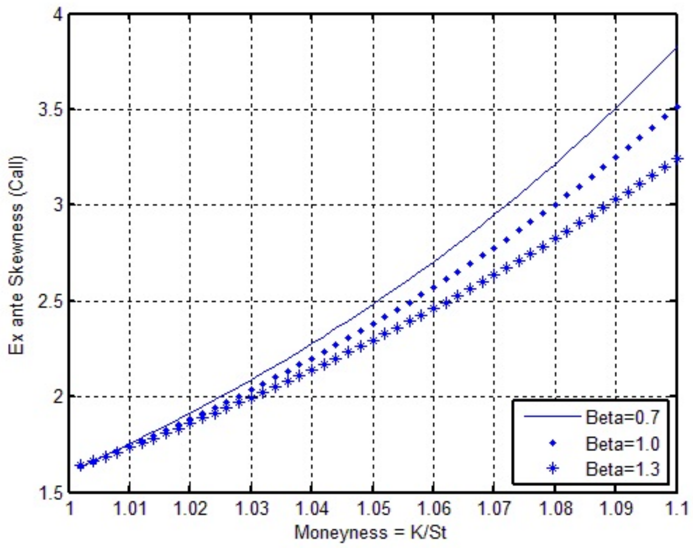
Panel B. Put Options



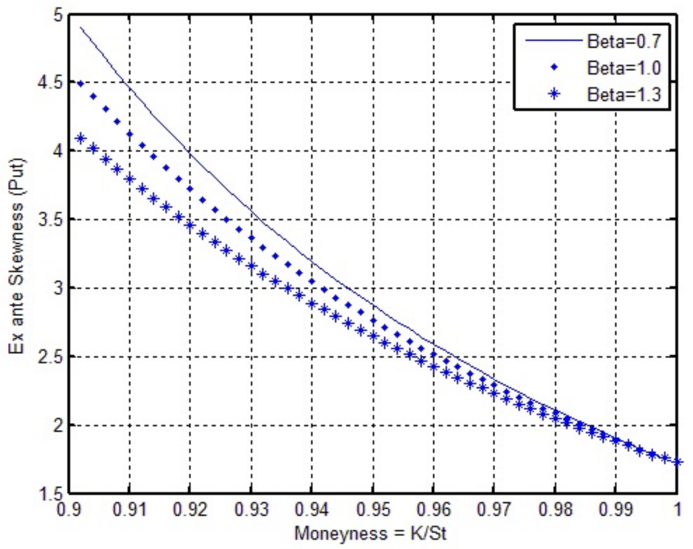
Note to Figure: This figure plots option ex ante skewness against moneyness (K/S). The parameters for the market index are $\kappa_I = 3.81$, $\theta_I = 0.0279$, $\delta_I = 0.356$, $\mu_I = 0.08$ and $\rho_I = -0.715$. The parameters for individual equity are $\kappa_j = 1.29$, $\theta_j = 0.042$, $\delta_j = 0.329$, $\alpha_j = 0.013$ and $\rho_j = -0.474$. The risk-free rate is set to 0.03. All the parameters are in annual basis.

Figure 3: Option skewness against Beta

Panel A. Call Options



Panel B. Put Options



Note to Figure: This figure plots option holding return skewness against betas. The parameters for the market index are $\kappa_I = 3.81$, $\theta_I = 0.0279$, $\delta_I = 0.356$, $\mu_I = 0.05$ and $\rho_I = -0.715$. The parameters for individual equity are $\kappa_j = 1.14$, $\theta_j = 0.0072$, $\delta_j = 0.128$, $\alpha_j = 0.01$ and $\rho_j = -0.656$. The risk-free rate is set to 0.03. All the parameters are in annual basis.

Table 1: Physical Parameters. Index and Equities

Ticker	Beta	Kappa	Theta	Delta	Rho	Alpha/Mu
SPX		2.83	0.0383	0.371	-0.855	0.056
JNJ	0.72	0.8	0.0219	0.187	-0.566	-0.006
KO	0.75	0.9	0.0252	0.213	-0.571	0.000
PG	0.78	0.85	0.0317	0.233	-0.346	0.006
MCD	0.78	1.01	0.0451	0.302	-0.426	0.017
WMT	0.81	0.54	0.0494	0.231	-0.549	0.010
PFE	0.89	0.96	0.0323	0.248	-0.574	0.020
MMM	0.91	0.99	0.0153	0.174	-0.478	-0.006
TRV	0.92	0.54	0.0256	0.16	-0.565	0.053
VZ	0.89	0.73	0.0323	0.217	-0.545	-0.020
UTX	0.91	1.04	0.0247	0.226	-0.376	0.078
MRK	0.92	1.28	0.033	0.291	-0.495	0.010
IBM	0.97	1.24	0.0126	0.177	-0.598	0.049
CVX	0.88	0.85	0.0272	0.078	-0.458	0.008
DD	0.99	0.76	0.0113	0.126	-0.542	0.000
T	0.97	0.52	0.0229	0.055	-0.434	-0.058
XOM	0.97	0.5	0.0267	0.008	0.297	0.007
BA	0.99	1.07	0.0323	0.263	-0.523	0.041
HPQ	1.06	1.29	0.042	0.329	-0.474	0.013
BAC	1.11	0.15	0.0159	0.068	-0.724	0.020
DIS	1.08	0.95	0.0119	0.15	-0.496	0.012
MSFT	1.11	0.99	0.0131	0.14	-0.523	0.036
CSCO	1.17	0.96	0.0586	0.333	-0.529	0.015
INTC	1.16	1.24	0.023	0.23	-0.492	0.053
CAT	1.16	0.87	0.006	0.102	-0.466	0.065
GE	1.11	0.99	0.0022	0.029	-0.561	-0.004
HD	1.16	1.04	0.0142	0.17	-0.611	0.035
AA	1.18	1.04	0.0135	0.14	-0.37	0.000
AXP	1.24	0.81	0.0018	0.054	-0.6	0.094
JPM	1.21	1.14	0.0072	0.128	-0.656	0.072

For the parameters that are theoretically constrained to be equal across measures, I use $\mathbb{Q}$-measure parameters estimated by Christoffersen, Fournier and Jacobs (2013). The physical parameters for index process are estimated by Chambers el al. (2014). The rest of the equity parameters and the spot volatility are estimated based on the methods discussed before.

Table 2: Average Hold-to-Expiration Put Returns

	Moneyness: K/St				
Ticker	0.92	0.94	0.96	0.98	1.00
JNJ	-0.88	-0.92	-0.42	-0.57	-0.38
KO	-0.68	-0.50	-0.39	-0.27	0.00
PG	-0.73	-0.64	-0.35	-0.35	-0.19
MCD	-0.41	-0.59	-0.24	-0.16	-0.12
WMT	-0.40	-0.54	-0.69	-0.33	-0.36
PFE	-0.61	-0.80	-0.43	-0.12	0.01
MMM	-0.53	-0.48	-0.79	-0.37	-0.30
TRV	0.35	-0.40	-0.17	-0.18	-0.56
VZ	-0.50	-0.72	-0.54	-0.30	-0.15
UTX	-0.82	-0.28	-0.37	-0.47	-0.12
MRK	-0.43	0.31	-0.46	-0.65	-0.52
IBM	-0.17	-0.36	-0.05	-0.25	-0.27
CVX	-0.22	-0.39	-0.29	-0.10	-0.09
DD	-0.53	-0.55	-0.42	-0.34	-0.25
T	-0.25	-0.48	-0.50	-0.51	-0.16
XOM	-0.64	-0.23	-0.54	-0.48	-0.27
BA	-0.69	-0.22	-0.39	-0.17	-0.07
HPQ	-0.56	-0.14	-0.21	-0.34	-0.12
BAC	-0.19	-0.50	-0.29	-0.53	-0.16
DIS	-0.55	-0.81	-0.79	-0.57	-0.37
MSFT	-0.85	-0.55	-0.59	-0.55	-0.34
CSCO	-0.79	-0.12	-0.29	-0.42	-0.40
INTC	-0.47	-0.37	-0.18	-0.29	0.01
CAT	-0.85	-0.38	-0.68	-0.32	-0.61
GE	-0.66	-0.71	-0.29	-0.15	-0.21
HD	-0.54	-0.64	-0.53	-0.47	-0.03
AA	0.15	-0.14	0.22	-0.87	-0.30
AXP	0.00	-0.45	-0.35	-0.32	-0.17
JPM	-0.35	-0.46	-0.55	-0.19	-0.32
Mean	-0.48	-0.45	-0.40	-0.37	-0.24

The table reports the population average hold-to-expiration returns for 29 equity put options divided into five moneyness groups, from 8% OTM to ATM.

Table 3: Average Sharpe Ratios for Put Returns

	Moneyness: K/St				
Ticker	0.92	0.94	0.96	0.98	1.00
JNJ	-5.30	-4.87	-0.22	-0.58	-0.39
KO	-0.56	-0.24	-0.24	-0.21	-0.02
PG	-0.66	-0.42	-0.16	-0.24	-0.18
MCD	-0.22	-0.35	-0.13	-0.13	-0.11
WMT	-0.22	-0.32	-0.86	-0.25	-0.32
PFE	-0.59	-1.70	-0.42	-0.10	-0.01
MMM	-0.26	-0.32	-1.06	-0.28	-0.28
TRV	0.09	-0.40	-0.09	-0.12	-0.70
VZ	-0.33	-0.87	-0.43	-0.23	-0.16
UTX	-1.25	-0.13	-0.22	-0.37	-0.11
MRK	-0.23	0.04	-0.31	-0.72	-0.49
IBM	-0.04	-0.20	-0.04	-0.14	-0.23
CVX	-0.10	-0.18	-0.14	-0.08	-0.09
DD	-0.27	-0.34	-0.33	-0.24	-0.21
T	-0.07	-0.39	-0.48	-0.60	-0.18
XOM	-0.48	-0.10	-0.42	-0.39	-0.22
BA	-0.61	-0.08	-0.32	-0.14	-0.08
HPQ	-0.42	-0.05	-0.14	-0.23	-0.09
BAC	-0.11	-0.39	-0.20	-0.51	-0.17
DIS	-0.35	-1.29	-1.01	-0.48	-0.33
MSFT	-1.71	-0.37	-0.55	-0.54	-0.32
CSCO	-0.74	-0.07	-0.20	-0.40	-0.37
INTC	-0.28	-0.27	-0.12	-0.19	-0.02
CAT	-1.53	-0.21	-0.91	-0.24	-0.73
GE	-0.76	-0.69	-0.15	-0.11	-0.17
HD	-0.22	-0.56	-0.44	-0.43	-0.03
AA	0.03	-0.11	0.11	-3.08	-0.27
AXP	-0.01	-0.32	-0.23	-0.21	-0.16
JPM	-0.18	-0.30	-0.52	-0.14	-0.32
Mean	-0.60	-0.53	-0.35	-0.39	-0.23

The table reports the population Sharpe ratios for 29 equity put options divided into five moneyness groups, from 8% OTM to ATM.

Table 4: Simulated Hold-to-Expiration Put Returns

	Moneyness K/St				
Ticker	0.92	0.94	0.96	0.98	1.00
JNJ	-0.57	-0.47	-0.38	-0.29	-0.22
KO	-0.53	-0.43	-0.34	-0.27	-0.21
PG	-0.59	-0.49	-0.40	-0.32	-0.24
MCD	-0.53	-0.44	-0.36	-0.29	-0.23
WMT	-0.52	-0.43	-0.35	-0.28	-0.23
PFE	-0.48	-0.41	-0.34	-0.28	-0.23
MMM	-0.50	-0.41	-0.33	-0.26	-0.20
TRV	-0.51	-0.44	-0.37	-0.31	-0.26
VZ	-0.51	-0.42	-0.34	-0.27	-0.21
UTX	-0.53	-0.45	-0.39	-0.33	-0.28
MRK	-0.51	-0.44	-0.36	-0.30	-0.25
IBM	-0.48	-0.41	-0.34	-0.29	-0.24
CVX	-0.59	-0.52	-0.43	-0.35	-0.29
DD	-0.44	-0.36	-0.29	-0.23	-0.19
T	-0.44	-0.36	-0.29	-0.23	-0.18
XOM	-0.59	-0.51	-0.44	-0.38	-0.32
BA	-0.46	-0.39	-0.33	-0.28	-0.23
HPQ	-0.35	-0.30	-0.26	-0.22	-0.19
BAC	-0.34	-0.29	-0.24	-0.20	-0.16
DIS	-0.38	-0.31	-0.26	-0.21	-0.17
MSFT	-0.43	-0.37	-0.31	-0.26	-0.22
CSCO	-0.36	-0.31	-0.27	-0.23	-0.19
INTC	-0.34	-0.30	-0.26	-0.22	-0.19
CAT	-0.43	-0.37	-0.31	-0.27	-0.23
GE	-0.41	-0.34	-0.28	-0.23	-0.18
HD	-0.41	-0.36	-0.30	-0.26	-0.22
AA	-0.29	-0.24	-0.20	-0.17	-0.14
AXP	-0.37	-0.32	-0.28	-0.24	-0.21
JPM	-0.39	-0.34	-0.30	-0.26	-0.22
Mean	-0.46	-0.39	-0.32	-0.27	-0.22

I use Monte Carlo simulation to compute the distribution of average returns for equity put options. I simulate $N = 10000$ times of index and 29 equities levels using Milstein scheme simulation. It is assumed that all risk premiums (except for the equity premium) are equal to zero.

Table 5: Simulated Sharpe Ratios for Put Returns

	MoneynessK/St				
Ticker	0.92	0.94	0.96	0.98	1.00
JNJ	-0.36	-0.31	-0.27	-0.24	-0.21
KO	-0.33	-0.29	-0.25	-0.22	-0.19
PG	-0.40	-0.35	-0.30	-0.26	-0.23
MCD	-0.36	-0.32	-0.28	-0.24	-0.22
WMT	-0.35	-0.31	-0.28	-0.24	-0.21
PFE	-0.32	-0.28	-0.26	-0.23	-0.21
MMM	-0.31	-0.27	-0.24	-0.21	-0.19
TRV	-0.35	-0.31	-0.29	-0.27	-0.24
VZ	-0.34	-0.30	-0.26	-0.23	-0.20
UTX	-0.36	-0.33	-0.30	-0.28	-0.26
MRK	-0.35	-0.32	-0.28	-0.26	-0.23
IBM	-0.32	-0.29	-0.26	-0.24	-0.22
CVX	-0.43	-0.39	-0.35	-0.31	-0.28
DD	-0.27	-0.24	-0.21	-0.19	-0.17
T	-0.28	-0.25	-0.22	-0.19	-0.17
XOM	-0.41	-0.38	-0.36	-0.33	-0.31
BA	-0.31	-0.28	-0.26	-0.24	-0.22
HPQ	-0.24	-0.22	-0.20	-0.19	-0.18
BAC	-0.22	-0.20	-0.18	-0.17	-0.16
DIS	-0.23	-0.21	-0.19	-0.17	-0.16
MSFT	-0.29	-0.26	-0.24	-0.22	-0.21
CSCO	-0.26	-0.24	-0.22	-0.20	-0.19
INTC	-0.23	-0.21	-0.20	-0.19	-0.18
CAT	-0.27	-0.26	-0.24	-0.22	-0.21
GE	-0.25	-0.23	-0.21	-0.19	-0.17
HD	-0.27	-0.25	-0.23	-0.22	-0.20
AA	-0.18	-0.17	-0.16	-0.15	-0.14
AXP	-0.23	-0.22	-0.21	-0.20	-0.19
JPM	-0.26	-0.24	-0.23	-0.22	-0.21
mean	-0.30	-0.27	-0.25	-0.23	-0.21

I use Monte Carlo simulation to compute the distribution of Sharpe ratios for equity put options. I simulate $N = 10000$ times of index and 29 equities levels using Milstein scheme simulation. It is assumed that all risk premiums (except for the equity premium) are equal to zero.

Table 6: Average Put Returns, Sharpe Ratios, and P-values

		Moneyness: K/St				
		0.92	0.94	0.96	0.98	1.00
	Data	-0.48	-0.45	-0.40	-0.37	-0.24
Returns	Model	-0.46	-0.39	-0.32	-0.27	-0.22
	P-value %	75.3	19.7	7.2	0.5	56.4
	Data	-0.60	-0.53	-0.35	-0.39	-0.23
Sharpe Ratios	Model	-0.30	-0.27	-0.25	-0.23	-0.21
	P-value %	11.8	12.9	5.8	10.6	42.3

The top panel of Table 6 summarizes the average returns for put options of 29 equities for various strikes. The *p*-values are computed based on the distributions of simulations and actual data. The distributions were constructed from 10,000 simulations for each of the stocks. The bottom panel reports the similar metrics for Sharpe Ratios.

Table 7: Ex Ante Skewness Measure

	Moneyness = 0.96		
Maturity	2 weeks	1 month	2 months
JNJ	3.57	2.88	2.49
KO	3.41	2.79	2.45
PG	3.56	2.90	2.53
MCD	3.19	2.67	2.38
WMT	3.12	2.62	2.33
PFE	3.08	2.62	2.37
MMM	3.34	2.78	2.47
TRV	3.11	2.68	2.46
VZ	3.11	2.60	2.30
UTX	3.26	2.81	2.61
MRK	3.12	2.64	2.38
IBM	3.07	2.66	2.47
CVX	3.42	2.83	2.51
DD	3.08	2.62	2.37
T	2.98	2.49	2.18
XOM	3.53	2.92	2.61
BA	2.95	2.57	2.37
HPQ	2.57	2.27	2.10
BAC	2.62	2.31	2.12
DIS	2.91	2.52	2.32
MSFT	2.89	2.53	2.36
CSCO	2.47	2.19	2.03
INTC	2.61	2.34	2.21
CAT	2.91	2.58	2.44
GE	2.93	2.53	2.32
HD	2.86	2.52	2.35
AA	2.54	2.25	2.08
AXP	2.78	2.50	2.40
JPM	2.74	2.46	2.34

This table illustrates the distribution of put option ex ante skewness for fixed moneyness (K/S = 0.96) with five different maturities. The ex ante skewness is computed based on the parameters from β-Heston model

Table 8: Average Returns and P-values

Skew Quintile	Days to Expiration 14			30			60		
	Dat	Sim	p-value	Dat	Sim	p-value	Dat	Sim	p-value
Low	-0.18	-0.19	0.90	-0.24	-0.22	0.54	-0.25	-0.25	0.98
2	-0.28	-0.25	0.48	-0.36	-0.26	0.01	-0.28	-0.28	0.94
3	-0.37	-0.32	0.43	-0.38	-0.31	0.09	-0.35	-0.32	0.63
4	-0.44	-0.43	0.87	-0.44	-0.38	0.19	-0.32	-0.36	0.51
High	-0.55	-0.60	0.18	-0.51	-0.48	0.64	-0.53	-0.44	0.15
Low-High	0.37	0.41		0.27	0.27		0.28	0.19	
(t-stat)	8.02	29.33		4.32	27.89		3.27	18.08	

Table 8 reports average portfolio returns from data and simulations for each ex ante skewness/maturity bin. The portfolios are constructed by sorting on ex ante skewness and the returns are Hold-to-Expiration returns, using the midpoint of the bid and ask prices as the proxy for price. The final two rows report differences in average returns across the high and low skewness quintiles along with t-statistics that test whether these differences are equal to zero.

Table 9: Table 9 Hold-to-Expiration delta-hedged return

	Total Volatility			Idiosyncratic Volatility		
Volatility Quintile	Dat %	Sim %	p-value	Dat %	Sim %	p-value
Low	-0.34	-0.74	0.02	-0.63	-0.82	0.03
2	-0.69	-0.88	0.97	-0.51	-0.96	0.62
3	-0.82	-0.94	0.55	-0.82	-0.99	0.39
4	-0.62	-1.17	0.09	-0.74	-0.98	0.76
High	-1.94	-1.18	0.00	-1.72	-1.14	0.00
Mean	-0.89	-0.82	0.71	-0.89	-0.85	0.68
Low-High	1.61	0.44		1.09	0.32	
(t-stat)	(5.04)	(12.68)		(2.96)	(9.65)	

Table 9 summarize the delta-hedged portfolio returns from actual data and simulations based on two sorting criteria: Total volatility and Idiosyncratic volatility. The portfolio is constructed by sorting the underlying total/idiosyncratic volatility. At the maturity of the put option, I rank the strategy returns into five quintiles based on the underlying idiosyncratic volatility (same procedures are repeated for total volatility). Note that the simulated portfolio returns are sorting based on total long-term volatility $\sqrt{\theta_j + \beta_j^2 \cdot \theta_i}$ and idiosyncratic long-term volatility $\sqrt{\theta_j}$.