Option-Based Measures of Co-Skewness and Co-Kurtosis Risk

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Abstract

I derive a new formula that expresses the measures of covariance, co-skewness, and co-kurtosis risk in terms of market risk-neutral moments and co-moments between stock and index returns. I then use the forward-looking information contained in the option prices to estimate option-implied moments and higher-moment correlations in order to construct market risk betas, co-skewness betas, and co-kurtosis betas. The empirical analysis suggests the following findings: compared to regression-based standard competitors, such as CAPM, the method that I have devised performs better in terms of mean squared error and R^2. An out-of-sample analysis of factor models incorporating co-skewness and co-kurtosis risk premium indicates that the new risk measures improve the return prediction. My results suggest that using option market information improves asset pricing in terms of model fit as well as out-of-sample forecasting power.

1 Introduction

The specification and estimation of factor models is of paramount importance for research and practice, and the methods of doing so are still debatable. The linear form of the risk-return relation suggested by the Capital Asset Pricing Model (CAPM) has been criticized based from different perspectives. The empirical findings in the literature contradict one of the fundamental principles in finance—that higher risk is associated with a higher expected return—posing as one of the major puzzles in finance literature. Consequently, there is a widespread consensus that models with better explanatory power are badly needed. Kraus and Litzenberger (1976) show that if investors care about portfolio skewness, co-skewness (which is the cornerstone of skewness) is relevant for asset pricing along with co-variation with the market portfolio. Similarly, if investors care about portfolio kurtosis, co-kurtosis is also relevant. I propose a new strategy to estimate the measures of co-skewness and co-kurtosis risk from option prices, along with the price of the corresponding risk, which can be used to forecast asset returns. The method is naturally forward-looking; thus, it avoids problems inherent to the use of cross-sectional regressions.

Risk premium is obviously a forward-looking concept. In essence, it compensates investors for holding an asset that will yield an uncertain return. In practice, however, the most commonly used method for estimating the risk premium is based on time series data. Conceptually, using historical excess return relies on the belief that noise will be canceled out in the long run. Thus, using historical risk premium is subject to the trade off between reflecting recent market condition and estimation accuracy. Merton (1980) argues that the historical risk premium fails to account for the effect of changes in the market risk.

Christofferssen et al. (2016) propose a new strategy to estimate the price of co-skewness and co-kurtosis risk from option prices that avoids problems inherent to the use of two-stage cross-sectional regressions. They show that the price of co-skewness risk corresponds to the spread between the physical and risk-neutral second moment, which is the market variance risk premium. In addition, the price of co-kurtosis risk is given by the spread between the physical and risk-neutral third moment, which is the market skewness risk premium. The information needed to pin down the price of risk comes from option prices, which are naturally time-varying and forward-looking. In simple terms, when moving into a volatile phase, investors are subject to higher uncertainty, and therefore a forward-looking risk premium should become higher. Then immediate price changes could be observed in the option market, while a historically-based estimation cannot be expected to reflect the changing market conditions.

Although it is appealing to consider co-skewness and co-kurtosis risk in cross-sectional asset pricing, there is no widespread consensus on their empirical relevance. This has been shown even in the simplest case when only covariance risk is considered : the CAPM has been heavily tested over the years; however, it has often been rejected. For instance, studies by Lakonishok and Sharpiro (1986), and Fama and French (1992) find no relation between market beta and average returns during the 1963-1990 period; further, recently, Baker, Bradley, and Wurgler (2011) show that high-beta stocks significantly underperform low-beta stocks.

Existing techniques for beta estimation use historical returns. These methods thus assume that the future will be sufficiently similar to the past, justifying simple extrapolation of current or lagged betas. However, no matter how sophisticated the modeling of time-variation in the betas, they are not able to capture sudden changes in the market. In this paper, I use option-implied information to estimate the exposure to co-skewness and co-kurtosis risk (namely, co-skewness beta and co-kurtosis beta, respectively). Option prices are inherently forward-looking and therefore contain valuable information of the future betas as opposed to the lagged ones. These measures can be computed using option data on a single day and, therefore, it is potentially possible to reflect sudden changes in the structure of underlying companies.

There is a growing amount of literature about extracting information from the option market to forecast asset returns and volatilities. Bakshi and Madan (2000) propose a model-free method to estimate the risk-neutral moments of underlying from option prices. For instance, Conrad, Dittmar, and Ghysels (2013) use option prices to estimate ex ante higher moments of risk-neutral returns distribution of underlying individual securities. They find that securities' risk-neutral volatility, skewness, and kurtosis are strongly related to the future returns. Many studies have demonstrated that option-implied volatility is a strong predictor of future volatility in equity markets. Classic contributions in this field include the ones from Christensen and Prabhala (1998), as well as Blair, Poon, and Taylor (2001). The predictive power of option-implied equity volatility has been confirmed recently by Busch, Christensen, and Nielsen (2008), who compare option-implied forecasts with state-of-the-art realized volatility forecasts.

To provide a completely forwarding-looking forecast of equity expected returns, we also need to estimate the corresponding risk premium. The most common way to do this is to calculate the average value of historical realized returns for a given period. However, it is very likely that the risk premium, reflecting a level of risk that is related to a different state of the world, will not occur later in the sample. Elton (1999) points out that future work in asset pricing should consider alternative ways to measure expected returns other than relying on the ex-post realized returns.

Chalamandaris and Rompolis (2016) propose another method to solve this issue. They extend the theoretical model of Duan and Zhang (2014) to a general system of equations that connects the cumulants of the physical distribution of any order to those of the risk-neutral cumulants through the projected relative risk aversion coefficient (PRRAC). Clearly, investors require a higher compensation to hold the market portfolio if a) they are more risk averse; b) they expect that future returns would be more volatile, more negatively skewed, and have a higher level of excess kurtosis. Another implication of their method is that it restricts the shape discrepancy between the physical and risk-neutral distributions by means of the PRRAC.

In this paper, I derive a new formula that expresses the measures of the co-variance, co-skewness, and co-kurtosis risk in terms of the risk-neutral moments of the market return and the higher order risk-neutral co-moments between the market and individual stock returns. I show that, compared to traditional regression-based methods, option-implied parameters perform well empirically. One source of the inputs to the formula—risk-neutral moments are computed directly from option prices; this is forward-looking and time varying. Another source of the inputs—risk-neutral co-moments are estimated by utilizing both information from stock return time series and the option market. Consequently, my approach has several distinctive features that separate it from conventional approaches.

First, because it is based on the current market prices instead of, for instance, accounting information, it can be implemented in real time. In principle, with the data available, we can update the parameters daily.

Second, my approach generates conditional forecasts at individual stock level. Rather than providing a vague unconditional average expected return on a portfolio of large value stocks, the new method is able to answer, for example, "what is the expected return on Microsoft today?".

Third, the parameters have specific, quantitative expressions that can be calculated using current market information. This is contrasted with factor models, in which both factor loadings and factors are estimated from time series data. The classical CAPM model requires forward-looking betas, which, in reality, are estimated based on historical data. My method provides a new perspective to solve this issue.

I empirically investigate the performance of my approach for the returns of individual stock and portfolio. Based on the monthly data for the period 2005-2014, I find that my option-implied forward-looking betas outperform regression-based estimates for both individual and portfolio returns. My results also indicate that the higher moment (co-skewness and co-kurtosis) risk premiums are priced in the cross-sectional asset returns. I further show that, with option-implied estimates for the higher moment risk premium, the predicted returns of portfolio are highly correlated with future realized ones.

This paper proceeds as follows. Section 2 gives a review of the method that Christofferssen et al. (2016) used to estimate the price of co-skewness and co-kurtosis risk. In Section 3, I derive the measures of co-skewness and co-kurtosis risk in terms of index returns risk-neutral moments and covariance, co-skewness, and co-kurtosis between individual equity returns and market index returns. Section 4 presents the estimation strategy for risk-neutral covariance, co-skewness, and co-kurtosis between equities and index returns. Section 5 provides the empirical results for the in-sample fit. Section 6 investigates the estimation of option-implied higher moment risk premium. Finally, Section 7 concludes.

2 The Price of Co-Skewness and Co-Kurtosis Risk

Absence of arbitrage implies the existence of a stochastic discount factor, m_{t+1}, that prices any asset with risky return, $R_{j,t+1}$, using the following condition:

$$E_t^P\left[(1 + R_{j,t+1})m_{t+1}\right] = 1 \tag{1}$$

where $E_t^P(.)$ denotes the expectation under the physical measure. Assume that the SDF can be written as a representative investor's marginal rate of substitution between current and future wealth. Under no arbitrage condition, the stochastic discount factor m_{t+1} must be nonnegative.

$$m_{t+1} = \frac{U'(W_{t+1})}{U'(W_t)}, \tag{2}$$

where $U'(.)$ is the marginal utility and W is the aggregate wealth. Since the marginal rate of substitution is not directly observable, to obtain testable restrictions from this first order condition, we usually define observable proxies for the marginal rate of substitution. Different proxies for the marginal rate of substitution and mechanisms have been proposed in different asset pricing models during the last four decades. Researchers use either observed returns of financial assets such as equity portfolios or non-market variables such as growth rate in aggregate consumption as the proxies for the marginal rate of substitution. Its form and specification is determined jointly by the assumptions about preferences and distribution of the proxies. As been pointed out by Harvey and Siddique (2000), the specification for the marginal rate of substitution can be viewed as a restriction on the set of trading strategies that the investors can use to maximize their utility.

Arrow (1971) argues that the desirable properties for an investors' utility function are (a) positive marginal utility for wealth, (b) decreasing marginal utility for wealth, and (c) non-increasing absolute risk aversion. The set of utility functions displaying these attributes are logarithmic, power and negative exponential utility functions. Since the exact form of utility function is unknown, they can be expanded as Taylor series:

$$m_{t+1} \approx h_0 + h_1 \frac{U''}{U'} R_{m,t+1} + h_2 \frac{U'''}{U'} R^2_{m,t+1} + ..., \tag{3}$$

where $R_{m,t+1}$ is the stock market return, which Christoffersen et al. (2016) used as a proxy for the return on the wealth portfolio. From equation (1), we have $E^P_t(m_{t+1}) = \frac{1}{1+R_{f,t}}$ for the risk-free rate, which gives

$$\frac{1}{(1+R_{f,t})} \approx h_0 + h_1 \frac{U''}{U'} E^P_t(R_{m,t+1}) + h_2 \frac{U'''}{U'} E^P_t(R^2_{m,t+1}) + ... \tag{4}$$

Combining equation (1) and (3), we can write the following form for the SDF

$$\begin{aligned} m_{t+1} = a_t + b_{1,t}\big(R_{m,t+1} - E^P_t(R_{m,t+1})\big) \\ +b_{2,t}\big(R^2_{m,t+1} - E^P_t(R^2_{m,t+1})\big) + b_{3,t}\big(R^3_{m,t+1} - E^P_t(R^3_{m,t+1})\big) \end{aligned} \tag{5}$$

equation(???) shows that the stochastic discount factor is cubic in the market return. The cubic form is consistent with investor's preference for higher order moments, such as skewness and kurtosis. In this case, the expected excess return will be related to co-kurtosis risk, apart from covariance risk and co-skewness risk. Dittmar (2002) explains that kurtosis measure the possibility of extreme values and co-kurtosis captures the sensitivity of asset returns to extreme market returns. if investors are averse to extreme values, they need compensation for holding co-kurtosis risk, in other word, the price of co-kurtosis risk should be positive.

As discussed by Harvey and Siddique (2000), in the traditional CAPM world, there are usually two routes: (a) A two-period world with homogeneous agents, where the representative agent's derived utility function (in wealth) is quadratic or logarithmic which guarantee that the discount factor is linear in the value-weighted portfolio of wealth; (b) Make distributional assumptions on the asset returns, which also keep the discount factor is linear in the value-weighted portfolio of wealth. The assumption that the SDF is linear in the market return produces the classic CAPM model.

Christoffersen et al. (2016) propose that in the absence of arbitrage, if the SDF has the form as equation (5), then the cross-sectional restriction on stock returns is

$$E_t^P(R_{j,t+1}) - R_{f,t} = \lambda_t^{MKT}\beta_{j,t}^{MKT} + \lambda_t^{COSK}\gamma_{j,t}^{COSK} + \lambda_t^{COKU}\delta_{j,t}^{COKU} \tag{6}$$

The prices of the *covariance* risk, λ_t^{MKT}, *co-skewness* risk λ_t^{COSK} and *co-kurtosis* risk, λ_t^{COKU}, are

$$\lambda_t^{MKT} = E_t^P(R_{m,t+1}) - R_{f,t}, \tag{7}$$

$$\lambda_t^{COSK} = E_t^P(R_{m,t+1}^2) - E_t^Q(R_{m,t+1}^2) \tag{8}$$

$$\lambda_t^{COKU} = E_t^P(R_{m,t+1}^3) - E_t^Q(R_{m,t+1}^3) \tag{9}$$

The model shows that the price of co-skewness risk is corresponding to the spread between the physical and the risk-neutral second moments of the market return. There is quite a number of studies in the literature for modeling the physical volatility of stock returns and estimating variance risk premium (e.g., Carr and Wu 2009). Bakshi and Madan (2006) also relate the volatility spread to risk aversion, Driessen, Maenhout and Vilkov (2009) study the price of correlation risk based on risk-neutral variance of index and its components.

Empirical studies conclude that the physical variance is lower than the risk-neutral variance, indicating a negative price of co-skewness risk (for instance, Bollerslev, Tauchen, and Zhou (2009), Bakshi and Madan (2006), and Jackwerth and Rubinstein (1996)). A negative price of co-skewness risk is intuitive: assets with lower co-skewness decrease the total skewness of the portfolio (more negative), and increase the probability of extreme losses. Thus, assets with lower co-skewness should have higher risk premium by the risk averse investors.

The price of co-kurtosis is equal to the spread between the risk-neutral and physical third moments. Current literature indicate that the risk-neutral distribution of index return is more left skewed than the physical ones, indicating a positive price of co-kurtosis risk. This is again consistent with theory, similar logic of covariance risk premium can apply here.

3 The Measures of Co-Skewness and Co-Kurtosis Risk

The usual way to estimate the measures of co-skewness and co-kurtosis risk is in a multivariate regression framework. However, in the following proposition, I show that these measures can be analytically solved by a system of linear equations, in terms of risk-neutral moments and co-moments of index and equity returns.

Proposition 1. *If the cross-sectional pricing restrictions are*

$$E_t^P(R_{j,t+1}) - R_{f,t} = \lambda_t^{MKT}\beta_{j,t}^{MKT} + \lambda_t^{COSK}\beta_{j,t}^{COSK} + \lambda_t^{COKU}\beta_{j,t}^{COKU}, \quad (10)$$

and the prices of the covariance *risk,* λ_t^{MKT}, co-skewness *risk* λ_t^{COSK} *and* co-kurtosis *risk,* λ_t^{COKU}*, are defined as*

$$\lambda_t^{MKT} = E_t^P(R_{m,t+1}) - R_{f,t}, \tag{11}$$

$$\lambda_t^{COSK} = E_t^P(R_{m,t+1}^2) - E_t^Q(R_{m,t+1}^2) \tag{12}$$

$$\lambda_t^{COKU} = E_t^P(R_{m,t+1}^3) - E_t^Q(R_{m,t+1}^3) \tag{13}$$

Then measures of the corresponding risk are solved by the following system linear equations

$$\begin{bmatrix} E^Q[(\tilde{R}_j - \bar{R}_j)(\tilde{R}_m - \bar{R}_m)] \\ E^Q[(\tilde{R}_j - \bar{R}_j)(\tilde{R}_m - \bar{R}_m)^2] \\ E^Q[(\tilde{R}_j - \bar{R}_j)(\tilde{R}_m - \bar{R}_m)^3] \end{bmatrix} = \begin{bmatrix} k_{m,2} & k_{m,3} & k_{m,4} \\ k_{m,3} & k_{m,4} - k_{m,2}^2 & k_{m,5} - k_{m,3} \cdot k_{m,2} \\ k_{m,4} & k_{m,5} - k_{m,2} \cdot k_{m,3} & k_{m,6} - k_{m,3}^2 \end{bmatrix} \cdot \begin{bmatrix} \beta_j^{MKT} \\ \gamma_j^{COSK} \\ \delta_j^{COKU} \end{bmatrix}$$

where $k_{m,n} = E^Q[(\tilde{R}_m - \bar{R}_m)^n]$.

The exact solution for betas can be found in Appendix A.

Proof. We could express equation (10) as (drop time subscript):

$$\begin{aligned} \tilde{R}_j - R_f = c_j + \beta_j^{MKT}\left(\tilde{R}_m - R_f\right) + \beta_j^{COSK}\left(\tilde{R}_m - \bar{R}_m\right)^2 \\ +\beta_j^{COSK}\left(\tilde{R}_m - \bar{R}_m\right)^3 + \tilde{e}_j \end{aligned} \tag{14}$$

where $\bar{R}_i$ denotes the $E^Q[R_i]$ for $i \in \{j, m\}$, and the zero-mean error term, $\tilde{e_j}$ is assumed to be independent of $\tilde{R}_m - R_f$, $(\tilde{R}_m - \bar{R}_m)^2$ and $(\tilde{R}_m - \bar{R}_m)^3$; Indeed, if we take expectation under physical measure of equation (14), write $[c_j]$ in terms of physical and risk-neutral moments of R_m, we could find the link (equivalence) between equation (10) and (14). Take the expectation of equation (14) under risk-neutral measure, subtracted from itself:

$$\begin{gathered}(10) - E^Q[(10)] \Rightarrow \tilde{R}_j - \bar{R}_j = \beta_j^{MKT}(\tilde{R}_m - \bar{R}_m) + \\ \gamma_j^{COSK}\{(\tilde{R}_m - \bar{R}_m)^2 - k_{m,2}\} + \delta_j^{COKU}\{(\tilde{R}_m - \bar{R}_m)^3 - k_{m,3}\}\end{gathered} \tag{15}$$

where $k_{m,n} = E^Q[(\tilde{R}_m - \bar{R}_m)^n]$, the n_{th}-moment under risk-neutral measure. Multiply equation (15) both sides by $(\tilde{R}_m - \bar{R}_m)$, and take the expectation under $\mathbb{Q}$:

$$E^Q[(\tilde{R}_j - \bar{R}_j)(\tilde{R}_m - \bar{R}_m)] = \beta_j^{MKT}k_{m,2} + \gamma_j^{COSK}k_{m,3} + \delta_j^{COKU}k_{m,4} \tag{16}$$

on the left side of equation (16), it is just the RN covariance between individual stock return R_j and market index return R_m. Repeat previous procedure, multiply equation (15) both sides by $(\tilde{R}_m - \bar{R}_m)^2$ and $(\tilde{R}_m - \bar{R}_m)^3$, respectively, then take the expectation, so that we have equation (17) and (18):

$$\begin{gathered}E^Q[(\tilde{R}_j - \bar{R}_j)(\tilde{R}_m - \bar{R}_m)^2] = \beta_j^{MKT}k_{m,3} + \\ \gamma_j^{COSK}(k_{m,4} - k_{m,2}^2) + \delta_j^{COKU}(k_{m,5} - k_{m,3} \cdot k_{m,2})\end{gathered} \tag{17}$$

on the left side of the equation (17), it is the risk-neutral co-skewness.

$$\begin{gathered}E^Q[(\tilde{R}_j - \bar{R}_j)(\tilde{R}_m - \bar{R}_m)^3] = \beta_j^{MKT}k_{m,4} + \\ \gamma_j^{COSK}(k_{m,5} - k_{m,2} \cdot k_{m,3}) + \delta_j^{COKU}(k_{m,6} - k_{m,3}^2)\end{gathered} \tag{18}$$

on the left side of the equation (18), it is the risk-neutral co-kurtosis. Combine equation (16), (17) and (18), we could express β_j^{MKT}, β_j^{COSK} and β_j^{COKU} in terms of risk-neural moments (which can be approximated by OTM option prices, see Appendix) and covariance, co-skewness and co-kurtosis. □

First to note that, as the pricing restriction (equation 10) is a natural extension to CAPM model, so is the measurement of corresponding risk. To see this, considering in a world when skewness and kurtosis risk is not priced, drop the high-moment related item in the matrix, we could have the expression for market Beta, $\beta_j^{MKT} = \frac{cov(R_j,R_m)}{var(R_m)}$. It is exactly the same as CAPM model.

The parameters $\beta_{i,t}^{MKT}$, $\beta_{i,t}^{COSK}$ and $\beta_{i,t}^{COKU}$, are functions of the market higher order moments (variance, skewness, kurtosis, etc), covariance, co-skewness and co-kurtosis, illustrate the relationship between my model and the Kraus and Litzenberger (1976) three-moment CAPM as well as Harvey and Siddique (2000) conditional skewness measure. It provides a testable restriction imposed on the cross section of asset expected returns from the asset pricing model incorporating skewness and kurtosis risk.

Empirical studies conclude that no one model solves the asset pricing puzzle and different combinations of factors work for different settings. Thus, in this paper, I proposed an asset pricing model that is a combination of the multifactor model with nonlinear components derived from asset return (co)skewness and (co)kurtosis. This is also consistent with Ghysels (1998) that nonlinear multi factor models behave better than linear beta model in the empirical studies.

3.1 General Case

Although there is relatively little research about the sign of terms in the SDF higher than third order. Christoffersen et.al (2016) give a more general nonlinearities form in the SDF, and its corresponding pricing model. if the stochastic discount factor (SDF) has the following form:

$$m_{t+1} = a_t + \sum_n b_{k,t}\big(G_n(R_{m,t+1}) - E_t^P[G_n(R_{m,t+1})]\big) + \sum_l c_{l,t}\big(f_{l,t+1} - E_t^P(f_{l,t+1})\big), \tag{19}$$

then the cross sectional pricing restrictions are

$$E_t^P(R_{j,t+1}) - R_f = \sum_n \lambda_t^n \beta_{j,t}^n + \sum_l \gamma_t^l \beta_{j,t}^l \tag{20}$$

and

$$E_t^P(R_{i,t+1}) - R_f = \sum_n \lambda_t^n \beta_{i,t}^n + \sum_l \gamma_t^l \beta_{i,t}^l, \tag{21}$$

where the β_t^n and β_t^l are from the projection of asset returns on $G_n(R_{m,t+1})$ and $f_{l,t+1}$ respectively, and γ^l is the price of risk associated with the factor f_l. The price of corresponding risk associated with market return λ_t^n, is

$$\lambda_t^n = E_t^P\big(G_n(R_{m,t+1})\big) - E_t^Q\big(G_n(R_{m,t+1})\big), \tag{22}$$

where $E_t^P(.)$ and $E_t^Q(.)$ denote the expectation under the physical and risk-neutral probability measure.

Proposition 2. *If the pricing restriction is in the form of equation (19), then the measures of the corresponding risk β can be calculated from following equation set, $k_{m,n} = E^Q[(\tilde{R}_m - \bar{R}_m)^n]$:*

$$\begin{bmatrix} E^Q[(\tilde{R}_j - \bar{R}_j)(\tilde{R}_m - \bar{R}_m)] \\ E^Q[(\tilde{R}_j - \bar{R}_j)(\tilde{R}_m - \bar{R}_m)^2] \\ \dots \\ E^Q[(\tilde{R}_j - \bar{R}_j)(\tilde{R}_m - \bar{R}_m)^n] \end{bmatrix} = \begin{bmatrix} k_{m,2} & k_{m,3} & k_{m,4} \\ k_{m,3} & k_{m,4} - k_{m,2}^2 & k_{m,5} - k_{m,3} \cdot k_{m,2} \\ \dots & \dots & \dots \\ k_{m,n} & k_{m,n+2} - k_{m,2} \cdot k_{m,n} & k_{m,k+3} - k_{m,3} \cdot k_{m,n} \end{bmatrix} \cdot \begin{bmatrix} \beta_j^{MKT} \\ \beta_j^{COSK} \\ \dots \\ \beta_j^{COn_{th}} \end{bmatrix}$$

Proof. The structure of the proof is similar to the proof of Proposition 1. The only extension is to multiply equation (15) both sides by $(\tilde{R}_m - \bar{R}_m)^n$, then apply the same procedure described in the proof of Proposition 1. □

4 Implied Moments and Correlation

In order to estimate the betas in equation (10), we need the conditional moments-co-moments (e.g., variance-covariance matrix) of the factors and vector of the conditional co-moments between the factors and the stock returns. The usual way to estimate these parameters is using historical stock returns. Nevertheless, what we really need the is the conditional moments-co-moments matrix, the use of historical data implies that the future is sufficiently similar to the past.

Instead of using solely the historical time series of the stock returns, we can estimate these moments from option prices, and use these information to construct the beta-parameters. There is a long history of using option implied information to forecast expected/future realized moments. For instance, the implied volatility of Black and Scholes (1973), or model free option implied moments of Britten-Jones and Neuberger (2000) and Bakshi, Kapadia, and Madan (2003).

The reason for using option implied information in the estimation is that the option prices subsume the current market expectations about future stock dynamics (e.g., Vanden (2008)), which is natural forward looking and time-varying. As shown by Blair, Poon and Taylor (2001), the implied volatility has better predictability for realized volatility in terms of R^2. Thus, using option-implied information in the parameters construction can potential increase their predictability.

Risk-neutral moments for stock and index can be easily computed from the observed option data. However, risk neutral co-moments pose a challenge, regarding to modeling and estimation. Buss and Vilkov (2012) propose a parameter way to estimate implied correlations such that the correlation matrix (a) meets all the necessary requirements of a correlation matrix (positive definite with absolute pairwise correlation smaller than one), (b) satisfies the identifying restriction that the weighted sum of index constituents implied variance is equal to the implied variance of the index. They use historical rolling window correlations (computed from daily and monthly returns) as input to the identification procedure of the implied correlations. In Appendix C, I illustrate the extended version of Buss and Vilkov (2012) method, to construct the estimates for the co-moments.

5 Data Description

This study is based on the major U.S. market proxy, the S&P100 index, and its constituents for period from January 4, 2005 to November 31, 2014. In section 5.1, I will briefly describe the stock and option data. In section 5.2, I introduce the estimation method for realized and option-implied measures.

5.1 Stock and Option Data

The daily stock data consist of prices (close price) and number of shares outstanding, plus S&P 100 index prices from *OptionMetrics* database. Sorted by cusip, there are total 203 names in the data, as the index constituents are changing through the time. The index weights are calculated using the closing market capitalization of all current index components on the previous day.

The data for equity and index options are also obtained from OptionMetrics Volatility Surface that provides Black-Scholes implied volatilities for options with standard maturities and moneyness level. As we are interested in most liquid options and also considering underlying stock investment horizon, I use options with approximately one month to maturity. I select out-of-the-money (OTM) as well as at-the-money (ATM) call and put options with this maturity. As the purpose of using options is not as instruments for trading, but as an information source only. Even if the OTM options are not so liquid some time, it is not be a big issue here.

5.2 Variance, Skewness and Kurtosis Estimation

I estimate the realized (co)variance, (co)skewness and (co)kurtosis as central moments from daily returns using the rolling window methodology with six months and one year window length.

For the risk-neutral moments, the model-free methodology is very welcome in the literature. Recent studies (e.g., Bakshi and Madan (2006), Carr and Lee (2009), Carr and Wu (2009)) show that the risk-neutral expected variance is best approximated by the model-free implied variance (MFIV).

Give the fact the MFIV method extracts information from all existing options expiring on one date and does not reply on any parametric model (while there is minor assumption on the stock process), I use this method to estimate all the risk-neutral moments needed in this paper (See Appendix for more detailed information). As pointed out by Carr and Wu (2006), the MFIV method is also chosen by practitioner to trade CBOE VIX.

5.3 Parameters Estimation

The empirical part of my study is to show that option-implied betas deliver a better result regarding asset pricing (measured by squared error or R-square). The reason behind this is that we managed to extract information about future dynamics from the option market as well as the information contained in the history of stock return time series.

From **Proposition 1**, we can have analytical closed form solution for betas as long as we have the following components: risk-neutral moments and risk-neutral co-moments. RN moments are estimated based on the Bakshi and Madan (2000) MFIM (model-free option-implied moments), more details can be found in Appendix. Estimating RN co-moments are divided into two parts: RN moments and RN correlations (include higher order correlations), again, RN moments get be extracted from option prices; while for the correlations, we need to build the bridge between the physical measure and the risk-neutral measure. Buss and Vilkov (2012) propose a semi-parametric way to model option-implied correlations, I extend their methods to higher order correlations.

[Insert Table 2 here]

Table 2 provides the summary statistics for market beta, co-skewness beta and co-kurtosis beta. The sample period spans from August 2005 to November 2014. For each month I compute three betas for all stock in the S&P100 index. The table reports the time-series median of these statistic. Additional, since option-implied method is able to capture the sudden change in the market, the ex-ante betas are more volatile then the ones from regression-based method, their distribution consequently is more skewed. Thus, I pool all the betas across time and stocks, and compute the 5%, 20%, 40%, 60%, 80% and 95% observation. Even tough these betas are calculated using option-implied information instead of estimated from regression based method, still the value range of β^{MKT}s is quite reasonable (For instance, the median of the market risk beta during the whole sample period is 1.08). The value for co-skewness betas and co-kurtosis betas are much larger, it is intuitive given the fact that the corresponding premium is much smaller in scale.

5.4 In-Sample Fitting

Once we have these forward looking parameters, if we are interested in the expected stock return, we still need corresponding risk premium. These risk premium, defined as the difference between the physical and the risk-neutral measure of the first three expected return moments. It is relatively straightforward to estimate the risk-neutral cumulants, using the model-free method proposed by Bakshi and Madan (2000). However, for the physical moments estimation, varies models/methods have been proposed to forecast the market premium and variance/volatility, there is no such consensus about which method to use. I will discuss this issue in the next section. At this section, we are only interested to see whether option-implied information is able to improve asset pricing, thus I run cross sectional regression to estimate the risk premium for each of the month. The sum of the squared residual is calculated and compared with the classical asset pricing model/method in the literature.

[**Insert Table 3 here**]

Table 3 shows the comparison between my method OiCCC (Option implied Measures of covariance-co-skewness-co-kurtosis risk) and other classical asset pricing models, $CAPM_{3Y}$ and $CAPM_{5Y}$ stand for CAPM model with three-year and five-year monthly moving window, respectively. $regCCC_{3Y}$ and $regCCC_{5Y}$ stand for regression based covariance-co-skewness-co-kurtosis with three-year and five-year monthly moving window, respectively. The CAPM and regCCC model are estimated using Fama-MacBeth two stage regression, I first use moving-window method to estimate the beta(s) for each stock, then a cross-sectional regression is run to determine the risk premium for each period: $R_{j,T} - r_f = c_j + \beta_j^{MKT}\lambda_T^{MKT} + \beta_j^{COSK}\lambda_T^{COSK} + \beta_j^{COKU}\lambda_T^{COKU}$. The sample period range from 2005.08 to 2014.11, with 112 month observation, cross-sectionally, we have 100 (S&P100 constituents, updated for each of the month) stocks for each month. After the OLSregression estimation, we have a 112*100 matrix of squared errorforeach method. Then for each month and each benchmark method, I compare the aggregate squared error (distribution) with the one from OiCCC, to see whether these benchmark methods significantly (at 10% level, t≥1.65) perform better/worse (measured by whether the MSE is higher or lower), and count the frequency. Panel A reports the average mean square error (MSE) for monthly returns, average cross-sectional R-square/adj. R-square and the frequencies that OiCCC outperforms the benchmark methods. Panel B reports the frequency that all the three premium (covariance-co-skewness-co-kurtosis) estimates are significant in the cross-sectional fit. Table 3 shows that the benchmark asset pricing methods very rarely outperform OiCCC while they are much frequently worse than OiCCC. More than quintile of the time, OiCCC performs significantly better than CAPM model, while the advantage shrinks when being compared to regCCC model. Panel B shows the frequency that all three premium estimates are significantly different from zero. Empirically, there is no such consensus about the significance of higher moment risk premium. Indeed, that is the case when we use regression based method to estimate the betas and then run cross-sectional regression to get the premium estimates. Panel B shows that there is only approximately 5% to 7% of the times when all three premium estimates are significant. Given this results, it is very natural to empirically reject the pricing models with higher moment risk premium. However, the frequency becomes much higher when using option-implied estimation method. Though they may not be high enough (for example, over 90% of the time), still with a frequency around 41% (with $p = 0.1$), it can be the supporting evidence that higher moment risk premium is priced in the cross-sectional asset returns.

5.5 Portfolio Analysis

Individual stock returns are relatively difficult to model, empirically, people are sometimes more interested in the portfolio return analysis/forecast. However, in my case, due to the technical difficult, I only study 100 stock returns for each time period, which means, in order to have sufficient number of observation for cross-sectional regression practice, the portfolio should be consist of 4 or 5 stocks.

In order to reflect the implied exposure of the corresponding risk for each stocks, I construct the following four portfolios: 1. portfolio sort by market beta; 2. portfolio sort by co-skewness beta; 3. portfolio sort by co-kurtosis beta; 4. portfolio sort by aggregate risk exposure. Take 1^{st} portfolio (sort by market beta) as an example, for each month, I sort the stocks based on their option-implied market beta, to form 20 portfolios. The first portfolio thereby contains the stocks with the lowest market betas, and the last portfolio contains the stocks with the highest market betas. The 4^{th} portfolio (sort by aggregate measures) is constructed differently. First, for each month, calculate the percentile of ranking for each beta, aggregate risk measure (ARM) is then defined as follows:

$$ARM = percentile(\beta^{MKT}) - percentile(\beta^{COSK}) + percentile(\beta^{COKU}) \quad (23)$$

Note that a negative sign on the co-skewness beta is also consistent with thoery, and with existing empirical studies that document a negative price of co-skewness risk (for instance, KL (1976) and HS (2000)). This portfolio sorting method is similar to the studies of Jensen, Black, and Scholes (1972), Baker, Bradley, and Wurgler (2011) and Buss and Vilkov (2012). For each portfolio, each month, and each methodology, I compute the equal-weighted expected portfolio risk measures (market beta, co-skewness beta and co-kurtosis beta) and realized portfolio return over the next month. Then I use cross-sectional regression to access the portfolio fit, measured by R-square.

[**Insert Table 4 here**]

Like the individual stock return case, I use three pricing models as the benchmark, the only difference here is that since we are evaluating the the model performance by the cross-sectional R-square, all of the benchmark models are estimated by two-stage Fama-Macbeth style regression (first betas, then premiums). Table 4 reports the mean R-square for each of the cross-sectional regression, for each of the methodology. As the table shows, for all portfolio construction methods, the OiCCC outperforms the other methods in a large scale. Overall, multi-factors models behave better, even after adjust the degree of freedom. OiCCC could relatively improve the adj.R-square by 100% comparing to other multi-factor models, and its improvement is consistent across different portfolios.

6 Estimating Physical Cumulants

In order to provide a completely forwarding-looking forecast of equity expected returns, we also need to estimate corresponding market premium. The most common way to do that is to calculate the average value for historical realized returns for a given period. Simple as it be, it has several problems: a) the average realized return is unconditional estimate. Given the fact that varies studies document premium (expected returns) being time-varying and persistent, indicating that the conditional ER is not very likely to be the same as the ex-post unconditional one. b) The long-run average does not take into account short-term changes in the market condition. It is very probable that the risk premium, reflects the level of risk which is related to the different state of the world will not occur in the sample later. Elton (1999) point out that future work in asset pricing should consider alternative ways to measure expected returns rather than relying on the ex-post realized returns.

There are different routes in the literature to solve this issue, for instance, use survey on academics or investors to get their view on the ER. Another literature use information from stock and option markets, together with a parametric option pricing model (Santa-Clara and Yan(2010)) or semi-parametric procedure (Duan and Zhang (2014)). Chalamandaris and Rompolis (2016) extend the theoretical model of Duan and Zhang (2014) to a general system of equations that connects the cumulants of the physical distribution of any order to those of the risk-neutral cumulants through the projected relative risk aversion coefficient (PRRAC).

The expected return of the market portfolio return is connected to higher-order physical cumulants and PRRAC. It is quite intuitive, investors require a higher compensation to hold the market portfolio if a) they are more risk averse; b) they expect the future returns would be more volatile, more negative skewed and higher level of excess kurtosis. Another implication of their method is that it restricts the shape discrepancy between the physical and risk-neutral distributions by means of the PRRAC.

6.1 Relationship Between Physical and Risk-Neutral Moments

Chalamandaris and Rompolis (2016) propose a semi-parametric relationship between physical and risk-neutral cumulants. Let $k_{t,n}^{\mathbb{Q}}$ and $k_{t,n}^{P}$ be the n^{th}-order cumulants of the τ period log-return $r_{t,T}$ distribution conditional on the current market information under the physical P and risk-neutral Q measure, respectively. Then, the relationship between physical cumulants and risk-neutral ones:

$$k_{t,n}^{P} = \sum_{m=0}^{\infty} k_{t,n+m}^{Q} \frac{\gamma^m}{m!}, \tag{24}$$

or in another form:

$$k_{t,n}^{Q} = \sum_{m=0}^{\infty} k_{t,n+m}^{P} \frac{(-\gamma)^m}{m!} \tag{25}$$

This is a general case of the well studied estimating physical moments from the risk-neutral ones during previous year. For instance, Bakshi, Kapadia and Madan (2003), Bakshi, Kapadia and Madan (2006) and Duan and Zhuang (2014) give approximate counterparts of equation (24) and equation (25) based on variance, skewness and kurtosis for n=3, n=2 and n=1, respectively. Their proposition provides a framework that express the physical distribution in terms of the risk-neutral ones and PRRAC γ. To see it intuitively, assume the log-return $r_{t,T}$ follows the normal distribution, which implies that $k_{t,n}^{P} = 0$ for $n > 2$, then the market risk premium will be

$$k_{t,1}^{P} - k_{t,1}^{Q} = \gamma k_{t,2}^{P} \tag{26}$$

which is a well-known result derived by CAPM and other models. It indicates that knowledge of γ and higher-order risk-neutral moments provides an estimate of the expected physical moments.

We can further express risk premium, which is defined as the difference between physical and risk-neutral measure, in the following way:

$$\lambda_n = k^{\mathbb{P}}_{t,n} - k^{\mathbb{Q}}_{t,n} = \sum_{m=1}^{\infty} k^{Q}_{t,n+m} \frac{\gamma^m}{m!}, \tag{27}$$

where λ_1 denotes the market risk premium, λ_2 and λ_3 denote price of the co-skewness and co-kurtosis risk, respectively.

6.2 Estimation of PRRAC coefficient

In order to estimate the PRRAC coefficient γ, we need to use the information contained in the equations of (24) and (25). For instance, if we set n=2, formula (24) and (25) lead to two similar expressions for the variance risk premium,

$$k^{P}_{t,2} - k^{Q}_{t,2} = \gamma k^{P}_{t,3} - \frac{\gamma^2}{2!} k^{P}_{t,r} + \frac{\gamma^3}{3!} k^{P}_{t,4} + ... \tag{28}$$

and

$$k^{P}_{t,2} - k^{Q}_{t,2} = \gamma k^{Q}_{t,3} - \frac{\gamma^2}{2!} k^{Q}_{t,r} + \frac{\gamma^3}{3!} k^{Q}_{t,4} + ... \tag{29}$$

The implication is interesting, it indicates that the variance risk premium can be attributed to higher-order (higher than second) physical/risk-neutral cumulants. As shown by several empirical researches, the negative variance risk premium can be explained by negative skewness. Following Bakshi and Madan (2006), we can implement a GMM method. Denote I_{t-1} as the information set known at time $t-1$. Then the orthogonality condition can be expressed as

$$E\Big[k^{P,Q}_{t,N} + \sum_{m=1}^{M} k^{P}_{t,m+2,m+N} \frac{(-\gamma)^m}{m!} | I_{t-1}\Big] = 0 \tag{30}$$

where $k^{P,Q}_{t,N} = \Big(k^{P}_{t,2} - K^{Q}_{t,2}, .., k^{P}_{t,N} - k^{Q}_{t,N}\Big)$ and $k^{P}_{t,m+2,m+N} = \big(k_{t,m+2}, .., k^{P}_{t,m+N}\ \big)$.

6.3 Physical Moments and Forward-Looking Premium

Using the estimates of γ along with higher-order physical and RN cumulants, we can calculate forward-looking market risk premium, co-skewness and co-kurtosis risk premium for each month. I implement third-order approximation of formula 24 and 25 to estimate the ex-ante risk premium. Table reports the descriptive statistics for the estimates of the moments and the forward-looking premium. First, the market risk premium is always positive, time-varying and counter-cyclical. It increases during the financial crisis period, the sub-prime mortgage crisis, as expected. On average the co-skewness risk premium is -0.0183 ($RA = 3$) and the co-kurtosis risk premium is 0.0022 ($RA = 3$). These results are consistent with theory and empirical results as well. In addition, it is important to mention that these existing estimates are usually mean of the price of risk over several years. Most of them use a two-stage Fama-MacBeth (1973) setting and report the average estimates of the monthly cross-sectional regression. It is likely that these prices of risk have the opposite sign over shorter time horizon. The advantage of this method is that we can have conditional monthly estimates of the price of risk that have the theoretically expected sign in almost every month.

[**Insert Table 5 here**]

6.4 Forecast Portfolio Return

In the previous section, I introduce the method for estimating the betas for the market risk, co-skewness risk and co-kurtosis risk, together with the corresponding risk premium estimated in this section, we can forecast conditional expected return for each stock and portfolio. I use the same method (as the one used in model fit) to sort the stocks according to their aggregate risk exposure. I then compute the equal-weighted expected portfolio betas (market beta, co-skewness beta and co-kurtosis beta) and multiply the price of the corresponding risk to get expected portfolio return for each month. I sort the portfolios into quintiles based on their expected return for each month, and compute the equal-weighted mean realized return for each quintile across all the time. The first quintile therefore contains the portfolios with highest expected returns, and the last one contains the portfolios with lowest expected returns. Additionally, I also use regression based method to estimate betas for CCC (covariance-co-skewness-co-kurtosis model), denote as regCCC, as a benchmark.

[**Insert Table 6 here**]

Table 6 reports the equal-weighted quintile return for different risk averse (RA) coefficient. As we can see, the option-implied method (OiCCC) generates a monotonic relation across different quintiles, and its performance is stable for different RA coefficients. While for regression based method, its relation is more noisy across different quintiles. The return difference between the extreme quintiles for OiCCC is about 10% (RA=4) annually, while it is only 3% for regCCC.

7 Conclusion

In this paper, I derive a new formula that expresses the measures of the co-variance, co-skewness and co-kurtosis risk in terms of the risk-neutral moments of the market return and the higher order risk-neutral co-moments between market and individual stock returns. Then I show that comparing to traditional regression based methods, these option implied parameters perform well empirically. I empirically investigate the performance of my approach for the individual stock and portfolio asset pricing. Based on the monthly data for the period 2005-2014, I find that my option-implied forward-looking betas outperform regression-based estimates, for both individual and portfolio returns. My results also indicate that the higher moment (co-skewness and co-kurtosis) risk premium is priced in the cross-sectional asset returns. I further show that, with an option implied estimates for the higher moment risk premium, the forecast portfolio return is highly correlated with future realized returns.

A Analytical Solution of Betas

$$\beta_t^{MKT} = -\frac{((E^{\mathbb{Q}}\big[(\tilde{R}_j - \bar{R}_j)(\tilde{R}_m - \bar{R}_m)^2\big] \cdot (k_{m,5} - k_{m,2} \cdot k_{m,3}) \cdot k_{m,4} - (k_{m,4} - k_{m,2}^2) \cdot E^{\mathbb{Q}}\big[(\tilde{R}_j - \bar{R}_j)(\tilde{R}_m - \bar{R}_m)^3\big] \cdot k_{m,4} - E^{\mathbb{Q}}\big[(\tilde{R}_j - \bar{R}_j)(\tilde{R}_m - \bar{R}_m)\big] \cdot (k_{m,5} - k_{m,2} \cdot k_{m,3})^2}{D}$$
$$+\frac{k_{m,3} \cdot E^{\mathbb{Q}}\big[(\tilde{R}_j - \bar{R}_j)(\tilde{R}_m - \bar{R}_m)^3\big](k_{m,5} - k_{m,2} \cdot k_{m,3}) - E^{\mathbb{Q}}\big[(\tilde{R}_j - \bar{R}_j)(\tilde{R}_m - \bar{R}_m)^2\big]k_{m,3} \cdot (k_{m,6} - k_{m,3}^2) + E^{\mathbb{Q}}\big[(\tilde{R}_j - \bar{R}_j)(\tilde{R}_m - \bar{R}_m)\big](k_{m,4} - k_{m,2}^2) \cdot (k_{m,6} - k_{m,3}^2)}{D} \tag{31}$$

$$\beta_t^{COSK} = -\frac{(-k_{m,4} \cdot E^{\mathbb{Q}}\big[(\tilde{R}_j - \bar{R}_j)(\tilde{R}_m - \bar{R}_m)^2\big] \cdot k_{m,4} \cdot +k_{m,3} \cdot E^{\mathbb{Q}}\big[(\tilde{R}_j - \bar{R}_j)(\tilde{R}_m - \bar{R}_m)^3\big] \cdot k_{m,4} + E^{\mathbb{Q}}\big[(\tilde{R}_j - \bar{R}_j)(\tilde{R}_m - \bar{R}_m)\big] \cdot k_{m,4} \cdot (k_{m,5} - k_{m,2} \cdot k_{m,3})}{D}$$
$$-\frac{k_{m,2} \cdot E^{\mathbb{Q}}\big[(\tilde{R}_j - \bar{R}_j)(\tilde{R}_m - \bar{R}_m)^3\big] \cdot (k_{m,5} - k_{m,2} \cdot k_{m,3}) - E^{\mathbb{Q}}\big[(\tilde{R}_j - \bar{R}_j)(\tilde{R}_m - \bar{R}_m)\big] \cdot k_{m,3} \cdot (k_{m,6} - k_{m,3}^2) + k_{m,2} \cdot E^{\mathbb{Q}}\big[(\tilde{R}_j - \bar{R}_j)(\tilde{R}_m - \bar{R}_m)^2\big] \cdot (k_{m,6} - k_{m,3}^2)}{D} \tag{32}$$

$$\beta_t^{COKU} = -\frac{(-k_{m,4} \cdot E^{\mathbb{Q}}\big[(\tilde{R}_j - \bar{R}_j)(\tilde{R}_m - \bar{R}_m)^2\big] \cdot k_{m,3} + E^{\mathbb{Q}}\big[(\tilde{R}_j - \bar{R}_j)(\tilde{R}_m - \bar{R}_m)\big] \cdot k_{m,4} \cdot (k_{m,4} - k_{m,2}^2) - E^{\mathbb{Q}}\big[(\tilde{R}_j - \bar{R}_j)(\tilde{R}_m - \bar{R}_m)\big] \cdot k_{m,3} \cdot (k_{m,5} - k_{m,2} \cdot k_{m,3})}{D}$$
$$+\frac{k_{m,2} \cdot E^{\mathbb{Q}}\big[(\tilde{R}_j - \bar{R}_j)(\tilde{R}_m - \bar{R}_m)^2\big](k_{m,5} - k_{m,2} \cdot k_{m,3}) + k_{m,3} \cdot k_{m,3} \cdot E^{\mathbb{Q}}\big[(\tilde{R}_j - \bar{R}_j)(\tilde{R}_m - \bar{R}_m)^3\big] - k_{m,2} \cdot (k_{m,4} - k_{m,2}^2) \cdot E^{\mathbb{Q}}\big[(\tilde{R}_j - \bar{R}_j)(\tilde{R}_m - \bar{R}_m)^3\big])}{D} \tag{33}$$

where

$$\begin{aligned} D = &-k_{m,4} \cdot (k_{m,4} - k_{m,2}^2) \cdot k_{m,4} + k_{m,3} \cdot (k_{m,5} - k_{m,2} \cdot k_{m,3}) \cdot k_{m,4} + k_{m,4} \cdot k_{m,3} \cdot (k_{m,5} - k_{m,2} \cdot k_{m,3}) \\ &-k_{m,2} \cdot (k_{m,5} - k_{m,2} \cdot k_{m,3})^2 - k_{m,3} \cdot k_{m,3} \cdot (k_{m,6} - k_{m,3}^2) + k_{m,2} \cdot (k_{m,4} - k_{m,2}^2) \cdot (k_{m,6} - k_{m,3}^2) \end{aligned} \tag{34}$$

and $k_{m,n} = E^{Q}[(\tilde{R}_m - \bar{R}_m)^n]$.

B Appendix: Model Free Option-Implied Moments

Bakshi, Carr and Madan (2000) show that any twice continuously differentiable fnct, $H(S_T)$, of terminal price S_T, can be replicated by a unique position in the risk-free, stocks and European options.

$$H[S] = H[\bar{S}]+(S-\bar{S})H_s[\bar{S}]+\int_{\bar{S}}^{\infty} H_{ss}[K](S-K)^+dK+\int_0^{\bar{s}} H_{ss}[K](K-S)^+dK \tag{35}$$

The prices of these contracts are

$$E_t^Q\{e^{-r\tau}H[S]\} = (H[\bar{S}] - \bar{S}H_s[\bar{S}])e^{-r\tau} + H_s[\bar{s}]S(t)+ \int_{\bar{s}}^{\infty} H_{ss}[K]C(t,\tau;K)dK + \int_0^{\bar{S}} H_{ss}[K]P(t,\tau;K)dK. \tag{36}$$

where $C_t(\tau, K)$ and $P_t(\tau, K)$ are prices of the European call and put options with time to maturity τ and strike price K. As a result, we can calculate the prices of derivatives given the price of the risk free zero coupon bond r, the spot price of the underlying, $\bar{S}$, and a series of OTM calls and puts. Since our main interest would be underlying return distribution, consider the function $H[S]$):

$$H[S_{t+\tau}] = R^2_{t+\tau} = (lnS_{t+\tau} - lnS_t)^2 \tag{37}$$

Then the risk-neutral variance, skewness and kurtosis of equity returns could be computed based on the following expressions.

$$E_0^Q\Big[e^{-rT}(\frac{S_T - S_0}{S_0})^2\Big] = \frac{2}{s_0^2}\Big[\int_0^{S_0} P_0(T,X)dX + \int_{S_0}^{\infty} C_0(T,X)dX\Big] \tag{38}$$

$$E_0^Q\Big[e^{-rT}(\frac{S_T-S_0}{S_0})^3\Big]=\frac{6}{S_0^2}\Big[\int_0^{S_0}(\frac{X-S_0}{S_0})P_0(T,X)dX+\int_{S_0}^{\infty}(\frac{X-S_0}{S_0})C_0(T,X)dX\Big] \tag{39}$$

$$E_0^Q\Big[e^{-rT}(\frac{S_T-S_0}{S_0})^4\Big]=\frac{12}{S_0^2}\Big[\int_0^{S_0}(\frac{X-S_0}{S_0})^2P_0(T,X)dX+\int_{S_0}^{\infty}(\frac{X-S_0}{S_0})^2C_0(T,X)dX\Big] \tag{40}$$

Since there is no continuity of strike prices, we can approximate the integrals using cubic spline. For a given maturity, I interpolate implied volatilities across different moneyness level (K/S) to obtain a continuum of implied volatilities. Furthermore, the implied volatility of the highest or lowest available strike price is used when moneyness below and above the available moneyness level in the market. More precisely, for moneyness level smaller than 1 ($K/S < 1$), the corresponding implied volatilities are used to generate put option prices, while for moneyness level larger than 1 ($K/S < 1$), the corresponding implied volatilities are used to generate call option prices.

C Option Implied co-skewness and co-kurtosis

Start from covariance, which is indeed correlation (since moments are "known"). Buss and Vilkov (2012 RFS) proposed an option-based measure for $\rho^Q_{ij,t}$

1. Consider market index I, with N components:

$$(\sigma^Q_{I,t})^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma^Q_{i,t}\sigma^Q_{j,t}\rho^Q_{ij,t} \tag{41}$$

2. They propose the following parametric form for implied correlations $\rho^Q_{ij,t}$:

$$\rho^Q_{ij,t} = \rho^P_{ij,t} - \alpha_t(1-\rho^P_{ij,t}) \tag{42}$$

3. Estimate $\rho^P_{ij,t}$ from historical rolling windows, then compute α_t, and identify $\rho^Q_{ij,t}$. Substitute the implied correlation (16) into restriction (15), one can compute α_t in closed form:

$$\alpha_t = -\frac{(\sigma^Q_{M,t})^2 - \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma^Q_{i,t}\sigma^Q_{j,t}\rho^P_{ij,t}}{\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma^Q_{i,t}\sigma^Q_{j,t}(1-\rho^P_{ij,t})} \tag{43}$$

4. In the end, we have the RN covariance:

$$Cov^Q_t(R_j, R_m) = \sigma^Q_{j,t}\sum_{i=1}^{N} w_i \sigma^Q_{i,t}\rho^Q_{ij,t} \tag{44}$$

Similarly, I propose the following strategy to estimate the co-skewness:

1. Define the central co-skewness (third moment):

$$\phi_{ijk} = E[(r_i - \bar{r}_i)(r_j - \bar{r}_j)(r_k - \bar{r}_k)], \tag{45}$$

   and skewness is a special case of co-skewness when $i = j = k$

2. The skewness of the market index:

$$\phi^Q_{mmm} = \sum_i \sum_j \sum_k w_i w_j w_k \phi^Q_{ijk} \tag{46}$$

3. Define *Skewness Correlation*

$$K_{ijk} = \frac{S_{ijk}}{\sqrt[2]{\phi_{iii}}\sqrt[4]{\phi_{jjj}}\sqrt[4]{\phi_{kkk}}} \tag{47}$$

4. Rewrite Equation (19):

$$\phi^Q_{mmm} = \sum_i \sum_j \sum_k w_i w_j w_k K^Q_{ijk} \sqrt[2]{\phi^Q_{iii}} \sqrt[4]{\phi^Q_{jjj}\phi^Q_{kkk}} \tag{48}$$

5. Impose parametric relationship between K^P_{ijk} and K^Q_{ijk}:

$$K^Q_{ijk,t} = K^P_{ijk,t} + \alpha_t(1 + K^P_{ijk,t}). \tag{49}$$

6. Estimate K^P_{ijk} from historical rolling data, identify the relationship parameter α_t from index skewness (substitute equation (23) into restriction (22)) and then compute K^Q_{ijk} accordingly. The expression for α_t

$$\alpha_t = \frac{\phi^Q_{mmm,t} - \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^N w_i w_j w_k \sqrt[2]{\phi^Q_{iii,t}} \sqrt[4]{\phi^Q_{jjj,t}\phi^Q_{kkk,t}} K^P_{ijk,t}}{\sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^N w_i w_j w_k \sqrt[2]{\phi^Q_{iii,t}} \sqrt[4]{\phi^Q_{jjj,t}\phi^Q_{kkk,t}}(1 - K^P_{ijk,t})} \tag{50}$$

7. Recall that on the left side of equation (13), we have the co-skewness between stock j and index m: $S^Q_{jmm} = E[(\tilde{R}_j - \bar{R}_j)(\tilde{R}_m - \bar{R}_m)^2]$, now becomes:

$$S^Q_{jmm} = \sqrt[2]{S^Q_{jjj}} \sum_i \sum_k w_i w_k K^Q_{ijk} \sqrt[4]{S^Q_{iii} S^Q_{kkk}}, \tag{51}$$

with each of the components on the right side of equation (24) estimated, the co-skewness S^Q_{jmm} is acquired.

The procedure for estimating co-kurtosis is analogous, just extend one more dimension.

1. Define the central co-kurtosis (fourth moment):

$$\psi_{ijkl} = E[(r_i - \bar{r}_i)(r_j - \bar{r}_j)(r_k - \bar{r}_k)(r_l - \bar{r}_l)], \tag{52}$$

   and kurtosis is a special case of co-skewness when $i = j = k = l$

2. The kurtosis of the market index:

$$\psi^Q_{mmmm} = \sum_i \sum_j \sum_k \sum_l w_i w_j w_k w_l \psi^Q_{ijkl} \tag{53}$$

3. Define *Kurtosis Correlation*

$$K_{ijkl} = \frac{\psi_{ijkl}}{\sqrt[4]{\psi_{iiii}}\sqrt[4]{\psi_{jjjj}}\sqrt[4]{\psi_{kkkk}}\sqrt[4]{\psi_{llll}}} \tag{54}$$

4. Rewrite Equation (26):

$$\psi^Q_{mmmm} = \sum_i \sum_j \sum_k \sum_l w_i w_j w_k w_l K^Q_{ijkl} \sqrt[4]{\psi^Q_{iiii}\psi^Q_{jjjj}\psi^Q_{kkkk}\psi^Q_{llll}} \tag{55}$$

5. Impose parametric relationship between K^P_{ijkl} and K^Q_{ijkl}:

$$K^Q_{ijk,t} = K^P_{ijk,t} - \alpha_t(1 - K^P_{ijk,t}). \tag{56}$$

6. Estimate K^P_{ijkl} from historical rolling data, identify the relationship parameter α_t from index kurtosis (substitute equation (23) into restriction (22)) and then compute K^Q_{ijkl} accordingly:

$$\alpha_t = \frac{\psi^Q_{mmmm,t} - \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^N \sum_{l=1}^N w_i w_j w_k w_l \sqrt[4]{\psi^Q_{iiii,t}\psi^Q_{jjjj,t}\psi^Q_{kkkk,t}\psi^Q_{llll,t}} K^P_{ijkl,t}}{\sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^N \sum_{l=1}^N w_i w_j w_k w_l \sqrt[4]{\psi^Q_{iiii,t}\psi^Q_{jjjj,t}\psi^Q_{kkkk,t}\psi^Q_{llll,t}}(1 - K^P_{ijkl,t})} \tag{57}$$

7. Recall that on the left side of equation (13), we have the co-kurtosis between

stock j and index m: $S^Q_{jmmm} = E[(\tilde{R}_j - \bar{R}_j)(\tilde{R}_m - \bar{R}_m)^3]$, now becomes:

$$\psi^Q_{jmmm} = \sqrt[4]{\phi^Q_{jjjj}} \sum_i \sum_k \sum_l w_i w_k w_l K^Q_{ijkl} \sqrt[4]{\phi^Q_{iiii} \phi^Q_{kkkk} \phi^Q_{llll}}, \tag{58}$$

with each of the components on the right side of equation (24) estimated, the co-kurtosis S^Q_{jmmm} is acquired.

D General Framework of Co-Moments

Suppose we have N assets and wish to determine the corresponding first three co-moments of the asset returns, i.e. the covariance of asset i and j:

$$\sigma_{i,j} = E\big[(R_i - \mu_i)(R_j - \mu_j)\big], \tag{59}$$

the products of three returns, i.e. the co-skewness of asset i,j and k:

$$\phi_{i,j,k} = E\big[(R_i - \mu_i)(R_j - \mu_j)(R_k - \mu_k)\big], \tag{60}$$

and similarly, we have the products of four returns, i.e. the co-kurtosis of asset i, j, k and l:

$$\psi_{i,j,k,l} = E\big[(R_i - \mu_i)(R_j - \mu_j)(R_k - \mu_k)(R_l - \mu_l)\big]. \tag{61}$$

We can also rewrite previous expression into matrix style: $N \times N$ covariance matrix Σ, $N \times N^2$ co-skewness matrix Φ and $N \times N^3$ co-kurtosis matrix Ψ of the corresponding return vector R with mean μ_R:

$$\Sigma = E\big[(R - \mu_R)(R - \mu_R)'\big] \tag{62}$$

$$\Phi = E\big[(R-\mu_R)(R-\mu_R)'\otimes(R-\mu_R)'\big]\,\Psi = E\big[(R-\mu_R)(R-\mu_R)'\otimes(R-\mu_R)'\otimes(R-\mu_R)'\big], \tag{63}$$

where $\otimes$ denotes the Kronecker product. For instance, if A is a $M \times N$ matrix and B is a $p \times q$ matrix, then the Kronecker product of A and B is the $mp \times nq$ block matrix:

$$\begin{bmatrix} A_{11}B & A_{12}B & ... & A_{1n}B \\ A_{21}B & A_{22}B & ... & A_{2n}B \\ . & . & . & . \\ . & . & . & . \\ A_{m1}B & A_{m2}B & ... & A_{mn}B \end{bmatrix}$$

Table 1: Number of Parameters

	Number of Elements
sigma	N(N+1)/2
phi	N(N+1)(N+2)/6
psi	N(N+1)(N+2)(N+3)/24
Total	N(N+1)/2+N(N+1)(N+2)/6 +N(N+1)(N+2)(N+3)/24
Total	
N=30	46,345
N=100	4,598,025
N=500	2,656,615,125,000,000

Furthermore, as mentioned by Peterson and Boudt (2008), we have the property that allow us to easily calculate the n-th portfolio/index return moment:

$$m_2(W) = E\big[\big(W'(R-\mu_R)\big)^2\big] = w'\Sigma w \tag{64}$$

$$m_3(w) = E\big[\big(W'(R-\mu_R)\big)^3\big] = w'\Phi(w \otimes w) \tag{65}$$

$$m_4(w) = E\big[\big(W'(R-\mu_R)\big)^4\big] = w'\Psi(w \otimes w \otimes w) \tag{66}$$

As been pointed out by previous studies, the challenge of this method is to estimate these co-moments/correlation. AS shown in Table , if we consider DJIA as market proxy, and study its components, we need to estimate $46,345$ parameters for each time period, which is quite acceptable. When we increase the number of components to 100, for instance, S&P 100 (which is my case), we would face $4,598,025$ parameters for each time period, under current hardware computing power, this is still acceptable. However, it might be a problem if we want to apply similar procedure to S&P 500 and its constituents.

E Regression Methods for Benchmark Comparison

I use those classical asset pricing models in the literature as benchmarks to compare the in-sample fitting performance, measured by squared error (SE) and R-square/adj. R-square. The models are CAPM and regression based covariance-co-skewness-co-kurtosis model (denote as regCCC, proposed by Christoffersen et. al (2016)) and Fama-French three factor model.

E.1 CAPM Model

I use Fama-MacBeth two stage regression to estimate the CAPM model. It has better explaining performance in the cross-sectional asset returns comparing to one-stage regression model. At first stage, for each stock j among 100 index components, I estimate the $\beta_{j,t}$ using moving window method based on following regression. The rolling windows I use is three-year and five-year monthly.

$$R_{j,t} - R_f = \alpha_j + \beta_{j,t} \times (R_{mkt,t} - R_f) + \epsilon_{j,t}. \tag{67}$$

Then regress all asset returns for each month (T) against the betas to determine the risk premium ($R_{mkt,T}$). For $j = 1$ to 100

$$R_{j,T} - r_f = \alpha_j + \beta_j \times (R_{mkt,T} - r_f) + \epsilon_{j,t}. \tag{68}$$

E.2 regCCC Model

Christoffersen et. al (2016) introduce this pricing model incorporating co-skewness and co-kurtosis risk apart from covariance risk. I also use Fama-MacBeth two stage regression to estimate this model. First, for each stock j, I estimate the market beta, co-skewness beta and co-kurtosis beta using moving window method based on following regression. The rolling windows I use is three-year and five-year monthly.

$$\tilde{R}_j - R_f = c_j + \beta_j^{MKT}\lambda_t^{MKT} + \beta_j^{COSK}\lambda_t^{COSK} + \beta_j^{COKU}\lambda_t^{COKU} \tag{69}$$

Where

$$\lambda_t^{MKT} = E_t^P(R_{m,t+1}) - R_{f,t}, \tag{70}$$

$$\lambda_t^{COSK} = E_t^P(R_{m,t+1}^2) - E_t^Q(R_{m,t+1}^2) \tag{71}$$

$$\lambda_t^{COKU} = E_t^P(R_{m,t+1}^3) - E_t^Q(R_{m,t+1}^3) \tag{72}$$

Then regress all asset returns for each month (T) against the betas to determine the risk premium. For $j = 1$ to 100

$$R_{j,T} - r_f = c_j + \beta_j^{MKT}\lambda_T^{MKT} + \beta_j^{COSK}\lambda_T^{COSK} + \beta_j^{COKU}\lambda_T^{COKU} \tag{73}$$

The expected physical moments are approximated by realized ones, while the risk-neutral moments are calculated based on model-free implied moments.

E.3 OiCCC model

OiCCC represents Option-implied covariance-co-skewness-co-kurtosis model. Instead of the estimating the betas based on historical moving window method. I use option-implied information to construct these risk measures. Thus, they are naturally forward-looking and time-varying. For a model in-sample fit testing, I use the same method as the Fama-macBeth second stage regression to determine the risk premiums:

$$R_{j,T} - r_f = c_j + \beta_j^{MKT}\lambda_T^{MKT} + \beta_j^{COSK}\lambda_T^{COSK} + \beta_j^{COKU}\lambda_T^{COKU}, \tag{74}$$

where the risk premium (λ) is defined as before.

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Table 2: Descriptive Statistic for Betas

Table 2 provides the summary statistics for market beta, co-skewness beta and co-kurtosis beta. The sample period spans from August 2005 to November 2014. For each month I compute three betas for all stock in the S&P100 index. The table reports the time-series median of these statistic. Additional, since option-implied method is able to capture the sudden change in the market, the ex-ante betas are more volatile then the ones from regression-based method, their distribution consequently is more skewed. Thus, I pool all the betas across time and stocks, and compute the 5%, 20%, 40%, 60%, 80% and 95% observation.

RA=3	Market Risk Beta	co-Skewness Beta	co-Kurtosis Beta
median	1.08	0.02	-17.25
5%	-3.65	-105.43	-6003.13
20%	-0.68	-24.69	-1185.98
40%	0.63	-4.85	-179.87
60%	1.56	4.38	127.77
80%	3.22	21.64	966.19
95%	8.54	64.38	3518.31

Table 3: Comparison with Benchmark Methods

Table 3 shows the comparison between my method OiCCC (Option implied Measures of covariance-co-skewness-co-kurtosis risk) and other classical asset pricing models, $CAPM_{3Y}$ and $CAPM_{5Y}$ stand for CAPM model with three-year and five-year monthly moving window, respectively. $regCCC_{3Y}$ and $regCCC_{5Y}$ stand for regression based covariance-co-skewness-co-kurtosis with three-year and five-year monthly moving window, respectively. The CAPM and regCCC model are estimated using Fama-MacBeth two stage regression, I first use moving-window method to estimate the beta(s) for each stock, then a cross-sectional regression is run to determine the risk premium for each period: $R_{j,T} - r_f = c_j + \beta_j^{MKT}\lambda_T^{MKT} + \beta_j^{COSK}\lambda_T^{COSK} + \beta_j^{COKU}\lambda_T^{COKU}$. The sample period range from 2005.08 to 2014.11, with 112 month observation, cross-sectionally, we have 100 (S&P100 constituents, updated for each of the month) stocks for each month. After the OLSregression estimation, we have a 112*100 matrix of squared errorforeach method. Then for each month and each benchmark method, I compare the aggregate squared error (distribution) with the one from OiCCC, to see whether these benchmark methods significantly (at 10% level, t$\geq$1.65) perform better/worse (measured by whether the MSE is higher or lower), and count the frequency. Panel A reports the average mean square error (MSE) for monthly returns, average cross-sectional R-square/adj. R-square and the frequencies that OiCCC outperforms the benchmark methods. Panel B reports the frequency that all the three premium (covariance-co-skewness-co-kurtosis) estimates are significant in the cross-sectional fit.

Panel A. Cross-Sectional Fit of Individual Stock Returns

	OiCCC	$CAPM_{3Y}$	$CAPM_{5Y}$	$regCCC_{3Y}$	$regCCC_{5Y}$
MSE	0.0049	0.0054	0.0054	0.0053	0.0053
R-square	0.1284	0.0454	0.0449	0.0815	0.0781
adj. R-square	0.1012	0.0354	0.0352	0.0528	0.0493
f. better than OiCCC		0	0	1/112	1/112
f. worse than OiCCC		23/112	24/112	12/112	17/112

Panel B. Summary of the Regression Statistics

	OiCCC	$regCCC_{3Y}$	$regCCC_{5Y}$
f. all three premiums are significant (10%)	41.1%	7.1%	7.1%
f. all three premiums are significant (5%)	34.8%	5.4%	6.3%

Figure 1: Difference between Mean Squared Error

Figure 1 plots the comparison between my method OiCCC (Option implied Measures of covariance-co-skewness-co-kurtosis risk) and other classical asset pricing models, $CAPM_{3Y}$ and $CAPM_{5Y}$ stand for CAPM model with three-year and five-year monthly moving window, respectively. $regCCC_{3Y}$ and $regCCC_{5Y}$ stand for regression based covariance-co-skewness-co-kurtosis with three-year and five-year monthly moving window, respectively. The CAPM and regCCC model are estimated using Fama-MacBeth two stage regression, I first use moving-window method to estimate the beta(s) for each stock, then a cross-sectional regression is run to determine the risk premium for each period: $R_{j,T} - r_f = c_j + \beta_j^{MKT}\lambda_T^{MKT} + \beta_j^{COSK}\lambda_T^{COSK} + \beta_j^{COKU}\lambda_T^{COKU}$. The sample period range from 2005.08 to 2014.11, with 112 month observation, cross-sectionally, we have 100 (S&P100 constituents, updated for each of the month) stocks for each month. After the OLSregression estimation, we have a 112*100 matrix of squared errorforeach method. Then for each month and each benchmark method, I compare the aggregate squared error (distribution) with the one from OiCCC, to see whether these benchmark methods significantly (at 10% level, t≥1.65) perform better/worse (measured by whether the MSE is higher or lower).

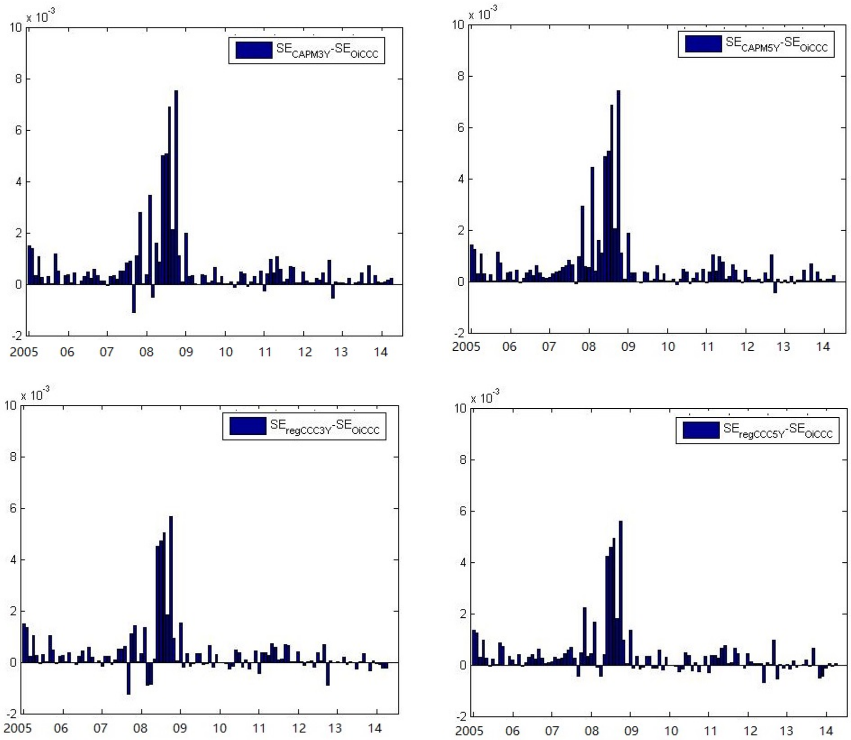


Table 4: Portfolio Fit Summary

I construct the following four portfolios: 1. portfolio sort by market beta; 2. portfolio sort by co-skewness beta; 3. portfolio sort by co-kurtosis beta; 4. portfolio sort by aggregate risk exposure. Take 1^{st} portfolio (sort by market beta) as an example, for each month, I sort the stocks based on their option-implied market beta, to form 20 portfolios. The first portfolio thereby contains the stocks with the lowest market betas, and the last portfolio contains the stocks with the highest market betas. For each portfolio, each month, and each methodology, I compute the equal-weighted expected portfolio risk measures (market beta, co-skewness beta and co-kurtosis beta) and realized portfolio return over the next month. Then I use cross-sectional regression to access the portfolio fit, measured by R-square (adj. R-square). The 4^{th} portfolio (sort by aggregate measures) is constructed differently. For each month, calculate the percentile of ranking for each beta, aggregate risk measure (ARM) is then defined as follows:

$$ARM = percentile(\beta^{MKT}) - percentile(\beta^{COSK}) + percentile(\beta^{COKU}) \tag{75}$$

Portfolio Fitting: Sort by β^{MKT}

	$CAPM_{3Y}$	$CAPM_{5Y}$	$regCCC_{3Y}$	$regCCC_{5Y}$	OiCCC
obervation	2800	2800	2800	2800	2800
MSE (%)	0.13	0.13	0.12	0.12	0.10
R-square (%)	8.50	8.72	19.43	18.49	25.94
adj. R-square (%)	4.53	4.75	7.92	6.85	15.36

Portfolio Fitting: Sort by β^{COSK}

	$CAPM_{3Y}$	$CAPM_{5Y}$	$regCCC_{3Y}$	$regCCC_{5Y}$	OiCCC
obervation	2800	2800	2800	2800	2800
MSE	0.15	0.15	0.13	0.13	0.10
R-square (%)	8.2	8.09	17.94	17.59	27.81
adj. R-square (%)	4.2	4.09	6.22	5.81	17.50

Portfolio Fitting: Sort by β^{COKU}

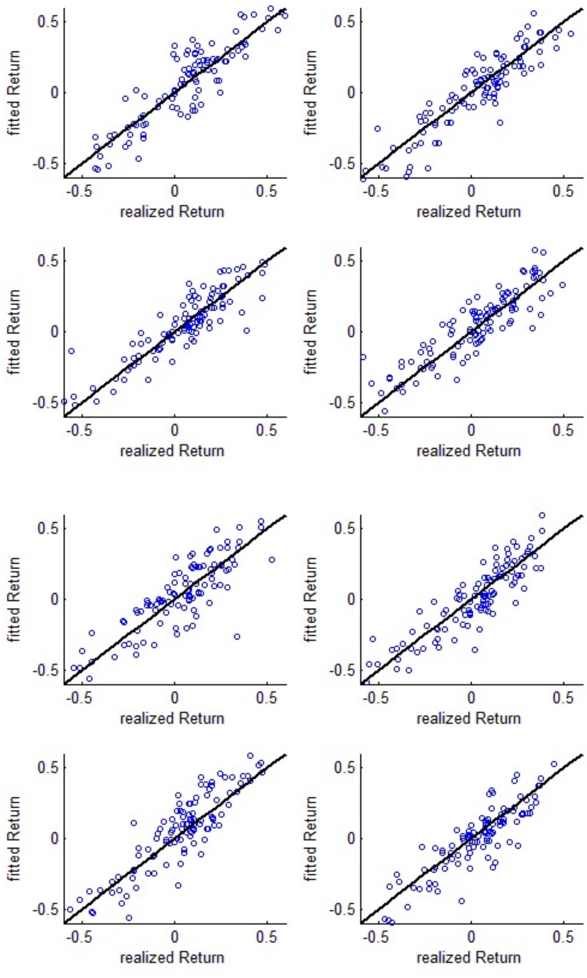
	$CAPM_{3Y}$	$CAPM_{5Y}$	$regCCC_{3Y}$	$regCCC_{5Y}$	OiCCC
observation	2800	2800	2800	2800	2800
MSE	0.14	0.14	0.12	0.12	0.10
R-square	9.25	9.51	18.89	20.04	26.92
adj. R-square	5.30	5.57	7.30	8.61	16.48

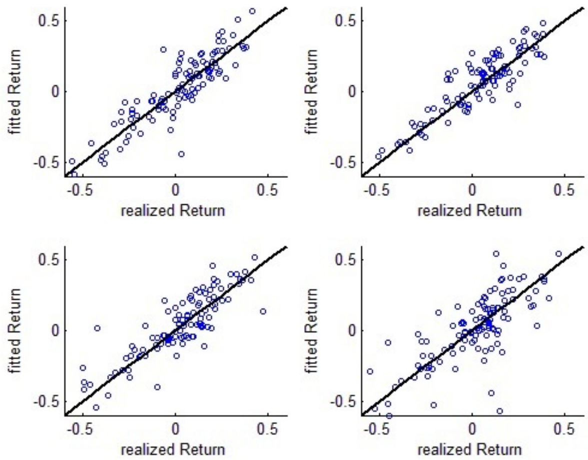
Portfolio Fitting: Sort by ARM

	$CAPM_{3Y}$	$CAPM_{5Y}$	$regCCC_{3Y}$	$regCCC_{5Y}$	OiCCC
observation	2800	2800	2800	2800	2800
MSE	0.14	0.14	0.12	0.13	0.10
R-square	8.65	9.10	19.62	18.11	27.47
adj. R-square	4.68	5.14	8.14	6.41	17.11

Figure 2: Scatter Plot between realized return and fitted-return

Figure 2 shows the scatter plot between realized portfolio returns and fitted-returns. For each portfolio, each month, I compute the equal-weighted expected portfolio risk measures (market beta, co-skewness beta and co-kurtosis beta) and realized portfolio return over the next month. Then I use cross-sectional regression to access the portfolio fit. The sample period is from August 2005 to November 2014.

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realized Return
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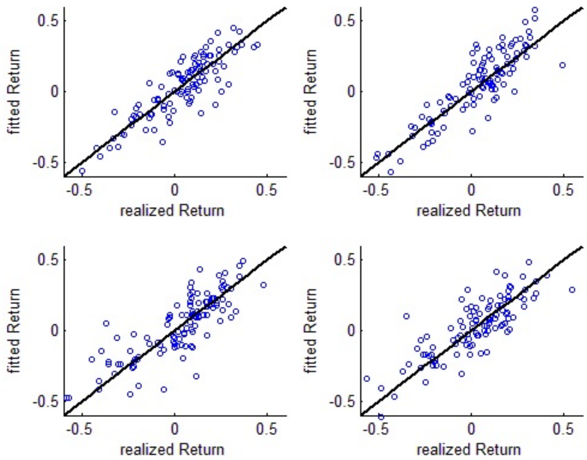
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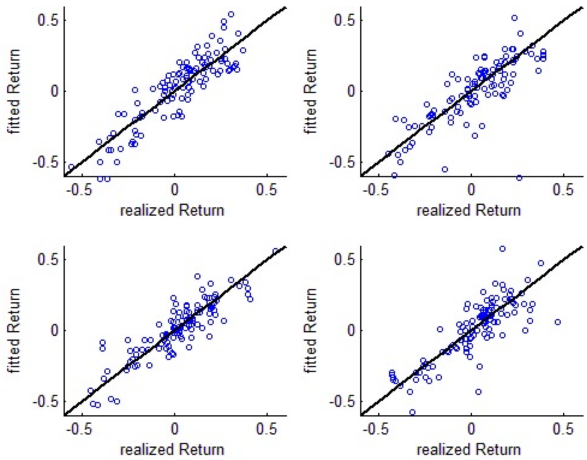
fitted Return
realized Return
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-0.5

Table 5: Price of Market Risk, Co-Skewness and Co-Kurtosis Risk

The table provides descriptive statistics for the market risk premium, the price of co-skewness and co-kurtosis risk, with different risk-aversion (RA) coefficient. The data are monthly and the mean and the standard deviation are reported in percentages. The risk premiums are estimated using the ex-ante option implied method of Chalamandaris and Rompolis (2016). The risk-neutral moments are estimated using the model-free approach in Bakshi and Madan (2000). The sample period is from August 2005 to November 2014.

RA=3	Market Risk	co-skewness Risk	co-Kurtosis Risk
mean	0.8192	-0.0183	0.0022
std	1.0783	0.0382	0.0031
skew	3.6101	-5.1791	1.9146
kurt	18.0276	33.0156	6.7769

RA=4	Market Risk	co-skewness Risk	co-Kurtosis Risk
mean	1.0819	-0.0233	0.0029
std	1.4164	0.0522	0.0039
skew	3.5824	-5.3883	1.9487
kurt	17.7942	34.9828	6.7542

RA=5	Market Risk	co-skewness Risk	co-Kurtosis Risk
mean	1.3408	-0.0279	0.0036
std	1.7449	0.0679	0.0048
skew	3.5505	-5.5785	2.0111
kurt	17.5307	36.8091	6.7983

The Figure 3, Figure 4 and Figure 5 plot the time series for the conditional market risk premium, price of the co-skewness and co-kurtosis risk, with different risk-aversion (RA) coefficient. The data are monthly and the mean and the standard deviation are reported in percentages. The risk premiums are estimated using the ex-ante option implied method of Chalamandaris and Rompolis (2016). The risk-neutral moments are estimated using the model-free approach in Bakshi and Madan (2000). The sample period is from August 2005 to November 2014.

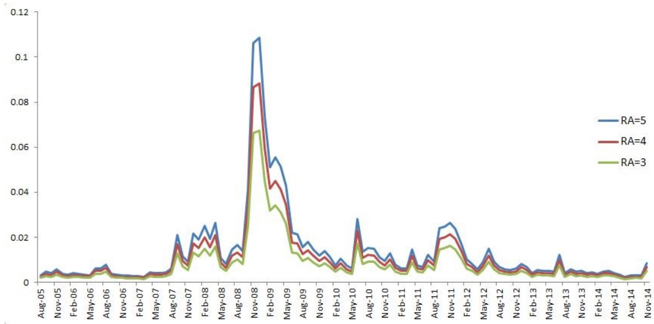

Figure 3: Market Risk Premium

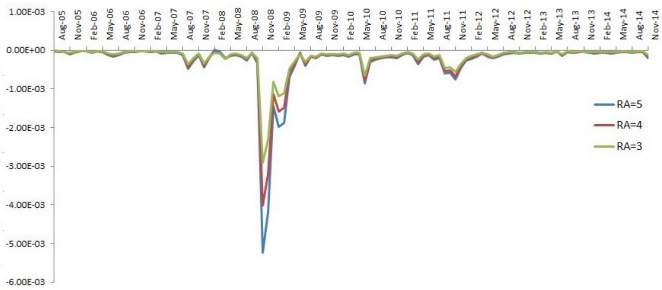

Figure 4: co-Skewness Risk Premium

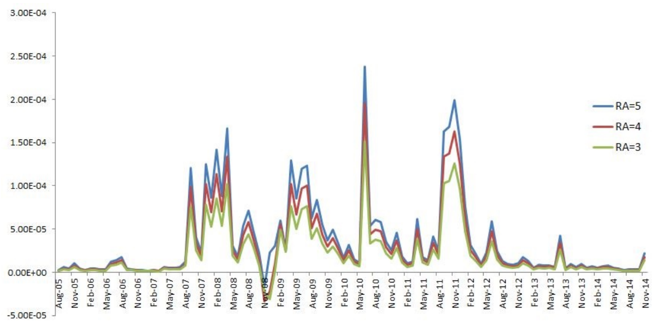

Figure 5: co-Kurtosis Risk Premium

Table 6: Out-of-Sample Portfolio return forecast

The betas are estimated by the option implied method introduced in this paper. The risk premiums are estimated based on the option-implied method of Chalamandaris and Rompolis (2016). I sort the stocks according to their aggregate risk exposure. I then compute the equal-weighted expected portfolio betas (market beta, co-skewness beta and co-kurtosis beta) and multiply the price of the corresponding risk to get expected portfolio return for each month. I sort the portfolios into quintiles based on their expected return for each month, and compute the equal-weighted mean realized return for each quintile across all the time. The first quintile therefore contains the portfolios with highest expected returns, and the last one contains the portfolios with lowest expected returns. Additionally, I also use regression based method to estimate betas for CCC (covariance-co-skewness-co-kurtosis model), denote as regCCC, as a benchmark. The sample period is from August 2005 to November 2014.

RA=3

Quintile	OiCCC	regCCC
1	0.1054	0.0618
2	0.0674	0.0593
3	0.0553	0.0525
4	0.0481	0.0770
5	0.0108	0.0346
1-5	0.0945	0.0273
p-value	0.0064	0.4608

RA=4

Quintile	OiCCC	regCCC
1	0.1063	0.0618
2	0.0707	0.0687
3	0.0592	0.0441
4	0.0442	0.0760
5	0.0066	0.0346
1-5	0.0996	0.0273
p-value	0.0043	0.4536

RA=5

Quintile	OiCCC	regCCC
1	0.1130	0.0657
2	0.0841	0.0595
3	0.0510	0.0475
4	0.0358	0.0765
5	0.0031	0.0359
1-5	0.1099	0.0298
p-value	0.0017	0.4112